# Combinatorics, Group Theory, and Topology at UNCG

in the

## Department of Mathematics & Statistics

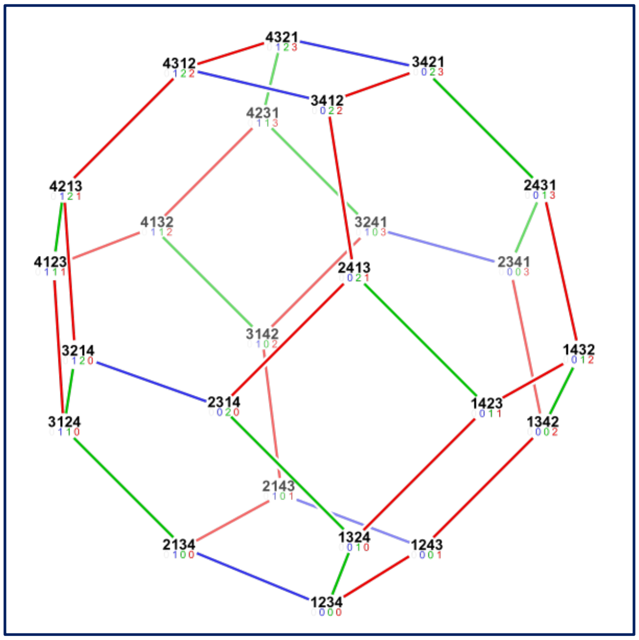
Combinatorics, Group Theory, and Topology are three active areas of research in pure mathematics at UNCG.

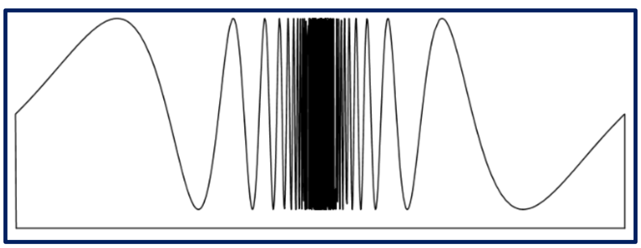
The Combinatorics Group works in combinatorial probability and combinatorial enumeration.

The modern study of infinite groups is comprised of studying their geometric, analytic, and of course algebraic structures. This interdisciplinary area is fairly young, and very active on an international scale.

UNCG’s topologists work with general and set-theoretic topology, geometric topology, topological algebra, and asymptotic topology.

Above right is the permutahedron of order 4; under that is a compactification of $\mathbb{R}$ with remainder equal to an interval.



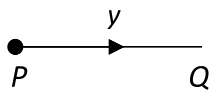



Mathematics Programs at UNCG

- Ph.D in Computational Mathematics
- M.A. in Mathematics
- B.S. and B.A. in Mathematics

Graduate teaching assistantships and tuition waivers are available.

## Group Theory

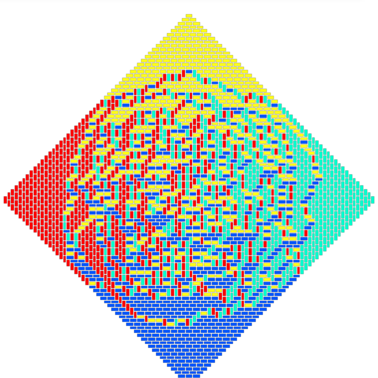



A group is the collection of symmetries of an object. In the study of group theory, there is an information exchange between a group and the object on which the group acts. The modern viewpoint is to consider the group's action on itself, where additional geometric or analytic structure has been imposed. The result is an interchange between the algebraic information of a group and its geometric and analytic counterparts. There is no computer algorithm that will determine whether two (finitely presented) groups are isomorphic, making the study of infinite groups computationally difficult. Group invariants therefore helps in distinguishing groups, as well as providing information about the spaces on which they act.

Dr. Talia Fernós
Associate Professor
Petty 143
t_fernos@uncg.edu

Dr. Fernós earned a Ph.D. in 2006 from the University of Illinois at Chicago and proceeded to do postdoctoral work at UCLA and Hebrew University in Jerusalem. She joined the UNCG faculty in 2010 and has also held short-term visiting positions at University of Neuchâtel in Switzerland, the Henri Poincaré Institute in Paris, France, and the University of Paris-Sud in Orsay France, and twice at the Mathematical Sciences Resaerch Institute in Berkeley, CA. Her research focuses on studying infinite groups via both geometric and analytic methods. Her work was recognized by an NSF fellowship and an NSF grant.

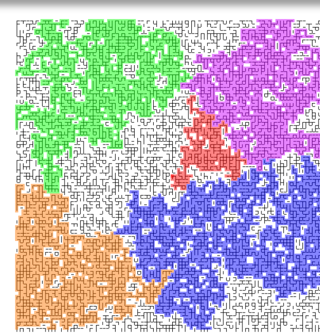
## Combinatorics



Combinatorics is the study of objects whose constituents are discrete, or separated, as opposed to those that are continuous. It is a vibrant field with major interactions with algebra, analysis, and probability and it has a substantive connection with almost every field in mathematics. There are dozens of overlapping areas of specialty within combinatorics, but graphs and hypergraphs, enumeration, and combinatorial geometry are particularly large and active categories of research.

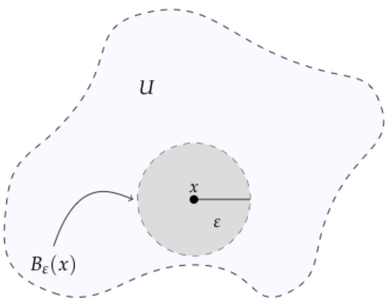
Dr. Cliff Smyth
Associate Professor
Petty 103
cdsmyth@uncg.edu

Dr. Smyth earned a Ph.D. in 2001 from Rutgers University and did postdoctoral work at IAS, CMU and MIT. He joined the UNCG faculty in 2008. His research lies in combinatorial probability, with forays into computational complexity, enumerative combinatorics, and other fields.



# Topology

Topology is the study of continuity. It is often described as a branch of geometry where two objects that can be continuously deformed to one another are considered to be the same. One goal of topology is to understand this notion of continuity in its essential form and to develop "invariants" that can help distinguish when two spaces are different. Invariants include the number of "connected" components, how many "holes" it has, or its "dimension." While this captures some of the spirit of topology, the reality is that the picture is more complex. Indeed, part of the study is to come up with pathological examples that do not behave as one might expect them to and to develop new invariants to help distinguish these spaces. The study is then an awe-inspiring safari exploring this plethora of wild examples.



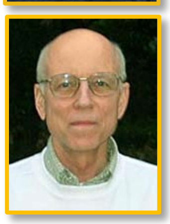

The basic topological notion of open set, shown here in a metric setting.

Dr. Greg Bell
Associate Professor
Petty 144
gcbell@uncg.edu

Dr. Bell received his Ph.D. in 2002 from the University of Florida under the direction of Alexander Dranishnikov. He was a VIGRE postdoc at Penn State and joined the UNCG faculty in 2005. His research studies coarse geometry, geometric topology, and geometric group theory. He currently serves as the Associate Dean of The Graduate School.

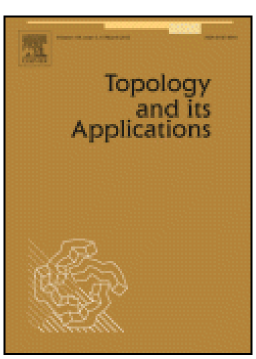
Dr. Yu-Min Chung
Assistant Professor
Petty 102
y_chung2@uncg.edu

Dr. Chung received his Ph.D. in Mathematics from Indiana University Bloomington in 2013 specializing in computational mathematics. He joined UNCG faculty in 2017. His main research focuses are Topological Data Analysis and applications to several scientific disciplines, including climatology, medical imaging, and biology.



Dr. Jerry Vaughan
Professor
Petty 111
vaughanj@uncg.edu

Dr. Vaughan received his Ph.D. from Duke University, and he joined the UNCG faculty in 1973. His research covers many areas in general and set-theoretic topology. His most recent work concerns the Stone-Čech compactifications of certain easily visualized topological spaces called $\psi$-spaces.

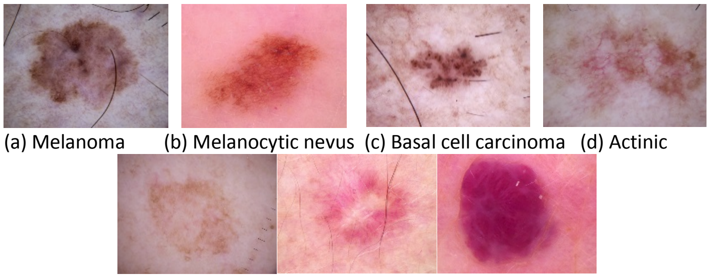



Topology and its Applications

Jerry Vaughan is a long time co-editor-in-chief of the international research journal "Topology and its Applications" published by Elsevier Science, and he co-edited two books in his specialty: "The Handbook of Set-theoretic Topology" published in 1984, and "The Encyclopedia of General Topology" in 2004.

# Research Activity at UNCG

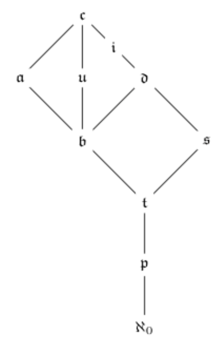
## Topology Data Analysis



(a) Melanoma (b) Melanocytic nevus (c) Basal cell carcinoma (d) Actinic
(e) Benign (f) Dermatofibroma (g) Vascular

Topological Data Analysis uses algebraic topology tools to uncover patterns and structure of given datasets. One of the projects Dr. Chung and his group have worked on is skin lesions analysis. Different types of skin lesions from the International Skin Imaging Collaboration are shown above. The main challenges are to investigate intrinsic information about each disease, and to classify different types of diseases.

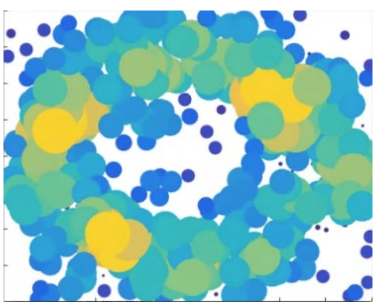
## Comparing Cardinals



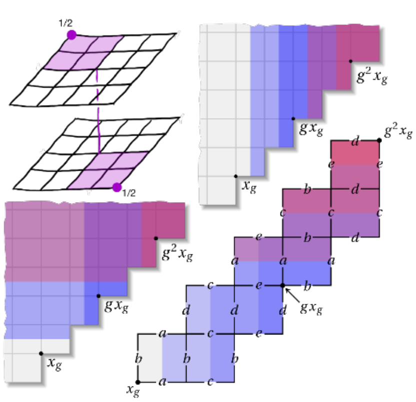

The diagram at the left is intended to display the basic relations among the ten cardinals shown there. Each cardinal is an uncountable cardinal not larger than the cardinality of the continuum $c$. Following the lines, if a cardinal $\kappa$ is lower on the diagram than cardinal $\lambda$, this indicates that there is a proof that $\kappa \leq \lambda$, and that there is no proof that they are equal (i.e., in some models of set theory $\kappa < \lambda$). If $\kappa$ and $\lambda$ are not related by line segments, then neither can be proved less than or equal to the other.

## Multi-radius Persistent Homology

This is the evolution of balls with radii proportional to the codensity. It can be seen that the region is covered more efficiently.



## Intervals in CAT(0) Cube Complexes

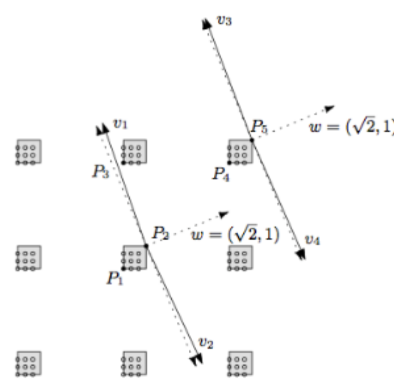


Intervals in CAT(0) cube complexes are key in understanding their structure and behavior at infinity. Intervals have been used to prove a Tits Alternative for groups acting on them, relationship with the Furstenberg-Poisson boundary, and superrigidity of irreducible lattices.

## Long Monotone Paths

Let $L = \{l_1, \ldots, l_n\}$ be a set of $n$ given lines in $\mathbb{R}^2$. A path in the arrangement A(L) is a simple polygonal chain joining a set of distinct vertices in $V = \{l_i \cap l_j : I < j\}$ by segments which are on lines in $L$. The length of a path is one plus the number of vertices in $V$ at which the path turns. A path is monotone in direction $(a, b)$ if its sequence of vertices is monotone when projected orthogonally along the line with equation $ay - bx = 0$. An interesting open question asks for the value of $\lambda$, the maximal monotone path length $N$ that can occur in an arrangement of $n$ lines. Clearly $\lambda(n) \leq \binom{n}{2} + 1$.

In [Balogh, Regev, Smyth, Steiger, Szegedy Discrete Comput Geom 32:167-176] the authors nearly settle the problem showing $\lambda(n) = \Omega\left(n^{2-\frac{d}{\sqrt{\log n}}}\right)$. The long monotone path and the line arrangement in which it lies are recursive in nature, the template for the kth level is shown.




### Contact Information

Contact math_sci@uncg.edu or visit us on the web at http://www.uncg.edu/math/.