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Some analogues of pair correlation of zeta zeros

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In 1972 H. L. Montgomery introduced the study of the pair correlation of zeros of the Riemann zeta-function $\zeta(s)$ and thereby opened a new direction for studying this function and the relations to the distribution of primes. His main motivation arose from a problem concerning the class numbers of imaginary quadratic fields. Montgomery's work, assuming the Riemann Hypothesis, not only gave results about the simplicity and distribution of the zeta zeros on the critical line, but also revealed connections to random matrix theory and was interpreted by Montgomery as being in accordance with the view (which legend dates back to Hilbert and Pólya) that there is a yet undiscovered linear operator whose eigenvalues characterize the zeros of $\zeta(s)$.

In our work, parts of it jointly with my doctoral student Yunus Karabulut, we first give an alternative way to develop Montgomery's argument. Our method has the advantage that it can also be applied in other instances for which we present some examples, viz. the correlation of zeta zeros with maxima points of $\zeta(s)$ on the critical line, the pair correlation of these maxima, and the correlation of zeros of one Dirichlet L -function with those of another Dirichlet L -function.

In my talk first I will describe Montgomery's work and its impacts on some subjects such as the theory of the Riemann zeta-function including relations with random matrix theory and the distribution of primes. Then I will recount those analogues of pair correlation of zeta zeros that we have worked out.