EXERCISES FOR DIOPHANTINE EQUATIONS

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1. Develop a method for computing all reducible monic cubic polynomials $g(t) \in \mathbb{Z}[t]$ of given discriminant Δ:
(i) Find a suitable transformation producing a polynomial $\tilde{g}(t)$ of the same discriminant but with $\tilde{g}(0) = 0$.
(ii) Show that $\tilde{g}(t) = t^3 + At^2 + Bt$ has discriminant $D_{\tilde{g}} = B^2(A^2 - 4B)$.

2. Compute all integral solutions of $y^2 = x^3 \pm 1$ by elementary methods.

3. Why is the computation of all solutions of a Thue equation $F(x, y) = m$ simpler if $F(x, y)$ is reducible?