Number Theory

in the

Department of Mathematics & Statistics

www.uncg.edu/mat/numbertheory

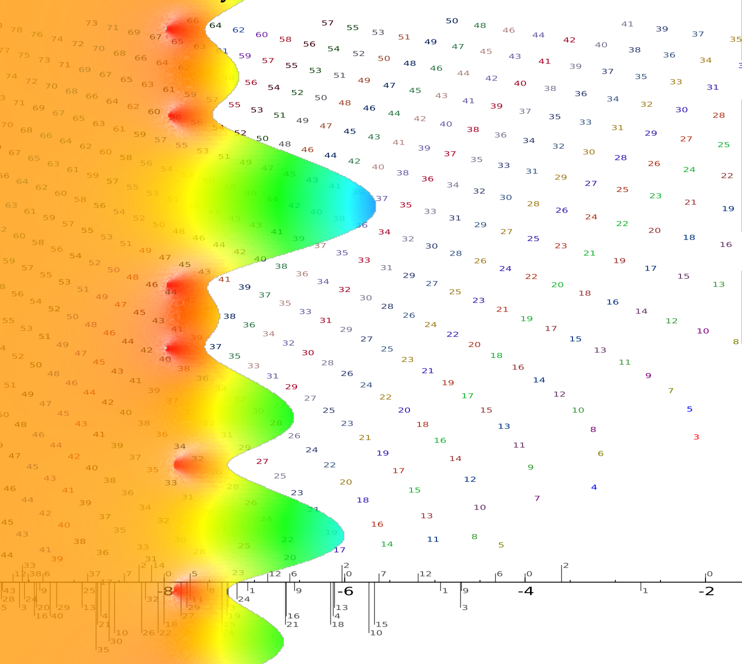
Number theory is one of the oldest research areas in mathematics. It is concerned with the study of integers (in particular prime numbers) and generalizations thereof.

On the right we give some examples from the long history of number theory. Some results took centuries to prove after they were first stated.

Public key cryptography is an important practical application of number theory in the everyday life of most people. Most secure digital communication is based on number theory and the security depends on the difficulty of solving number theoretic problems.

The members of the number theory group at UNCG work in several areas, including algebraic, analytic, and computational number theory, and the theory of modular forms.

300BC	Euclid proves there are infinitely many primes and proposes the Euclidean Algorithm for computing greatest common divisors.
1600S	Fermat proves that $a^{p-1} \bmod p = 1$ for all primes p, and integers a not divisible by p (Fermat's little theorem). He states that $a^n + b^n = c^n$ has no solution for positive integers a, b, c, if $n > 2$ (Fermat's last theorem).
1859	Riemann hypothesizes that the non-real zeros of the zeta function $\zeta(s)$ (pictured on the left) all have real part $\frac{1}{2}$. This important conjecture remains unproven to this day.
1896	Hadamard and de la Vallée-Poussin independently employ $\zeta(s)$ in proofs of the prime number theorem that states that the function $\pi(n)$ giving the number of prime numbers up to n is asymptotic to $\frac{n}{\log n}$.
1977	Rivest, Shamir, and Adleman invent the RSA public key cryptosystem. It uses Fermat's little theorem.
1995	Wiles proves Fermat's last theorem.



The graph on the far left shows the absolute value of the Riemann zeta function $\zeta(\sigma + it)$ for $0 \leq \sigma \leq 6$ and $-58 \leq t \leq -0.3$. The zeros are the white dots in the valleys.

The other plot gives the distribution of the zeros of higher derivatives of $\zeta(s)$ in the left half plane.

The Number Theory Faculty

Dr. Sebastian Pauli
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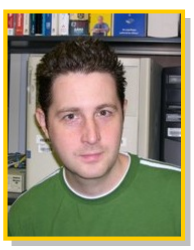
Dr. Pauli wrote his Diplomarbeit (Masters thesis) under the supervision of Michael Pohst at Technische Universität Berlin. He received his Ph.D. from Concordia University in Montreal in 2001 and as a Post Doctoral Fellow was the lead developer of the computer algebra system KASH/KANT during the years 2002–2006. He has been at UNCG since 2006.

His research is in computational number theory. He is particularly interested in algorithms for local fields and computational class field theory. He is also investigating the distribution of the zeros of the derivatives of the Riemann Zeta function.

Computational Number Theory

This area of mathematics deals with developing algorithms for explicit computation of the objects under consideration and the implementation of these algorithms. Using these methods, abstract objects are better understood and many new examples can be found. These can then be used to further develop the theory.



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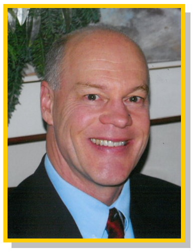
Dr. Saidak received his B.Sc. at The University of Auckland in New Zealand, and his M.Sc. and Ph.D. in number theory at Queen's University in Ontario, Canada. He then held postdoctoral and visiting positions at the University of Calgary (Alberta), University of Missouri (MO), Macquarie University (Sydney), and Wake Forest University (NC).

He is mainly interested in classical questions concerning prime numbers and their distribution, which he investigates using analytic and probabilistic methods. A special topic of his interest is the location of zeros of the Riemann zeta function and its derivatives; others include problems centering around the differences between consecutive primes, distribution and divisibility properties of primes of special forms, as well as values of various arithmetical functions.

Analytic Number Theory

Analytic number theory is a branch of number theory that employs methods and techniques from complex analysis in order to investigate arithmetic properties of the integers and the distribution of prime numbers. The Prime Number Theorem (1896) is the central result of this subject, while the Goldbach Conjecture (1742) and the Riemann Hypothesis (1859) are the most famous open problems.



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Dr. Tangedal earned his Ph.D. from the University of California at San Diego in 1994 under the direction of Harold Stark. After holding various positions at the University of Vermont, Clemson University, and the College of Charleston, he joined the faculty at UNCG in 2007.

His research interests lie in algebraic number theory with a particular emphasis on explicit class field theory. This involves the constructive generation of relative abelian extensions of a given number field using the special values of certain transcendental complex and p-adic valued functions. Almost all of his research to date is concerned with a system of conjectures, due to Stark and others, that make class field theory explicit in a precise manner using the special values mentioned above.

Algebraic Number Theory

Algebraic numbers were first studied in a systematic way by Gauss and Kummer in order to formulate and prove the laws of higher reciprocity in number theory. The study of algebraic number theory for its own sake, and for its connections to such areas as algebraic geometry and cryptography, forms a vast domain of current research.

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Dr. Yasaki has an M.A. (2000) and Ph.D. (2005) from Duke University under the supervision of L. Saper. After a three year post-doc at the University of Massachusetts, he has been part of the UNCG faculty since 2008. In 2009, he was a Visiting Scholar at the University of Sydney, where he developed a computer package for computing Bianchi cusp forms in Magma.

His research interests are in the area of modular forms, particularly the connection between explicit reduction theory of quadratic forms and the computation of Hecke data for automorphic forms. Recent work has focused on producing new examples of cusp forms over number fields of small degree.

Modular Forms

'Modular forms are functions on the complex plane that are inordinately symmetric. They satisfy so many internal symmetries that their mere existence seem like accidents. But they do exist.' (Barry Mazur)

The theory of modular forms was first developed in connection with the theory of elliptic functions in the first half of the 19th century. They show up in many different areas of mathematics, revealing connections that are still not fully understood.

UNCG Summer School in Computational Number Theory

www.uncg.edu/mat/numbertheory/summerschool

Since 2012, the number theory group at UNCG has organized annual 1-week summer schools — hosting an average of 30 participants each year. This project is funded by the National Security Agency, the National Science Foundation, and the Number Theory Foundation. Further summer schools are planned for the coming years.

The summer school aims to complement the traditional training that graduate students receive by exposing them to a constructive and computational approach to objects in number theory.

Posters for previous UNCG Summer Schools in Computational Number Theory.

On a typical day of the summer school, external and local experts give talks in the morning and in the afternoon students solve problems related to this material. The talks early in the week introduce the students to the subject. Talks later in the week cover related areas of current research and unsolved problems.

Mathematics Programs at UNCG

- Ph.D. in Computational Mathematics
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- B.S. in Mathematics
- B.A. in Mathematics

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