A HAWK-DOVE GAME IN FINITE KLEPTOPARASITIC POPULATIONS

S. GRUNDMAN, L. KOMÁRKOVÁ, AND J. RYCHTÁŘ

Abstract. Kleptoparasitism, the stealing of food items, is a widespread biological phenomenon. In this paper we extend earlier models of kleptoparasitism to investigate a finite population of individuals involved in foraging and, potentially, kleptoparasitism. We assume that the population consists of two types of individuals, Hawks and Doves. Hawks use every opportunity to steal and they also defend their food items. Doves never resist and never steal. We derive and study the stochastic model of this kleptoparasitic population. We compare the stochastic model with the deterministic model of a Hawk-Dove game in kleptoparasitic populations as well as with the abstract version of the game. We demonstrate that the outcome of the model depends upon various natural parameters, for example food density, the handling time of a food item, the size of the population, as well as the duration of potential fights over the food. The exact dependence on parameter values is much richer in stochastic description than in the deterministic version.

1. Introduction

Kleptoparasitism (parasitism by theft) is the stealing of prey that has been caught by another individual, [27]. In a broader perspective, it is a form of feeding where one individual takes prey from another that has caught, killed, or otherwise prepared it. The most common observations of kleptoparasitism have been amongst birds [3, 4, 13]; however it has been observed and documented across a great diversity of taxa such as large carnivorous mammals [10, 19], lizards [11], fish [15, 16], insects [18, 26], snails [17], and spiders [1].

There is now a substantial literature of works using game theoretic models to investigate kleptoparasitic behavior in nature (e.g. [2, 30, 8, 28, 9]). Almost all of the models are deterministic, based on a certain system of ordinary differential equations and thus implicitly assuming an infinitely large population of individuals. However, model organisms that exhibit kleptoparasitism either live in or are observed in relatively small populations and thus are likely to violate assumptions of infinite population size models. To our knowledge, only two papers to date ([23] and [32]) proposed stochastic models for finite kleptoparasitic populations.

In this paper, we extend earlier models of kleptoparasitism from [8] and [32] to investigate a finite population of conspecifics involved in foraging and, potentially, kleptoparasitism. We assume that the population consists of two types of individuals, Hawks and Doves. Hawks use every opportunity to steal and they also defend

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their food items. Doves never resist and never steal. We derive and study the stochastic model of the population. Using Gillespie's algorithm [14], we perform the MCMC simulations. We conclude that the outcome of the dynamics depends upon various natural parameters, for example food density, the handling time of a food item, the size of the population, as well as the duration of potential fights over the food. We compare the stochastic model with the deterministic model as well as with the classical version of the abstract Hawk-Dove game.

2. The classical Hawk-Dove game

The Hawk-Dove game (also known as the game of chicken [25]) was introduced by John Maynard Smith and George Price in [22]. It is used to describe a situation in which there is competition for a shared resource and the contestants can choose either an aggressive (Hawk) or non-aggressive (Dove) strategy for the competition. Mathematically, it is a two player zero sum matrix game with the payoff matrix usually given by

	Hawk	Dove
Hawk	$V/2 - C/2$	V
Dove	0	$V/2$

where V is the value of the contested resource, and C is the cost of an escalated fight (see [20, 21]). It is (almost always) assumed that the value of the resource is less than the cost of a fight, i.e.,

$$C > V > 0$$

The entries of the matrix reflect the fact that when Hawk encounters Hawk, one of them will get a resource of value V whereas the second will bear the cost of the fight C (or, equivalently, both will bear their share of a cost of the fight $C/2$). When Hawk meets Dove, Hawk gets the whole resource while Dove gets nothing. Finally, when Dove meets Dove, they "split".

It can be calculated (see e.g. [20]) that, if $C > V > 0$, then the population tends to an equilibrium where the fraction of Hawks in the population equals to $\frac{V}{C}$. If $C < V$, then Hawk only population is the stable equilibrium.

3. Deterministic model of kleptoparasitic Hawk-Dove game

The basic structure of our model follows and extends the deterministic models introduced in [8, 6]. Individuals forage for food, and can be in one of the following four states:

- *searcher* – looking for food, but having yet to find it;
- *handler* – having found food, preparing to consume it;
- *challenger* – having come across a handler, trying to steal the food item;
- *defender* – trying to resist the challenge of another.

We assume that individuals take an exponential time, with mean t_h, to handle a food item, and that the food is consumed instantly at the end of this period.

We consider a population consisting of the two types. A type is determined by the reaction of an individual to an encounter with another, where one of the two birds is handling food, and the other has an opportunity to try to steal it.

Hawk	always attacks, always resists when attacked
Dove	never attacks, never resists when attacked

Table 1. The model parameters and their definitions (top section) and notation (bottom section).

Symbol	Meaning
ν_f	rate that individuals search for food items
f	food density
ν_h	rate that individuals search for handlers
t_h	expected handling time (if unchallenged)
$\frac{t_a}{2}$	expected duration of a contest over food
α	probability that the challenger wins the contest
N	number of Hawks in the (finite) population
π	chances that an average Hawk will do better than an invading Dove
H	total number of Hawks
H_s	number (or density, for deterministic model) of searching Hawks
H_h	number (or density, for deterministic model) of handling Hawks
D	total number of Doves
D_s	number (or density, for deterministic model) of searching Doves
D_h	number (or density, for deterministic model) of handling Doves

Each individual is initially searching for food items. We assume that they find food at rate $\nu_f f$ (where f is the density of food). At the same time, Hawks are searching for handlers as well (at rate ν_h).

When a Hawk finds a handler, it challenges and tries to steal its food. If the handler is a Dove, it will surrender the food item (and has to search again for another one). But if the handler is a Hawk, it will defend its food and they will fight for it. The challenger wins the fight with probability $\alpha \in (0, 1)$.

If the fights are only between Hawks, i.e. at the end of the fight it will always be a Hawk that will win (and will start handling) and a Hawk that will lose (and will start searching for another food item), all of the results presented here are independent of α and we assume that $\alpha = 1/2$. The choice $\alpha = 1/2$ allows the most natural interpretation of the results that follows, as it means that it takes, on average, 2 fights for a Hawk to win and obtain food. The fights take a random time with exponential distribution with mean $\frac{t_a}{2}$. We assume that the only cost of the fight is the time spent in the contest.

Also, since the fights are between Hawks only, we can simplify the structure of the model by merging challengers and defenders into fighters (once the individuals enter the fight, there is no real distinction between them). All of the parameters are summarized in Table 1.

3.1. **Conclusions of the deterministic model.**

The deterministic model for infinite population was studied in [6]. The core of the paper was to analyze a system of differential equations (for densities of individuals engaged in certain activity) based on the diagram in Figure 1.

The fitness, i.e. how well a certain strategy does is measured by how much time an individual using that strategy spends in the handling state. For an individual type, its fitness is proportional to the fraction (density of handling individuals)/(total density of individuals), see [6]. In the setting of this model, the value

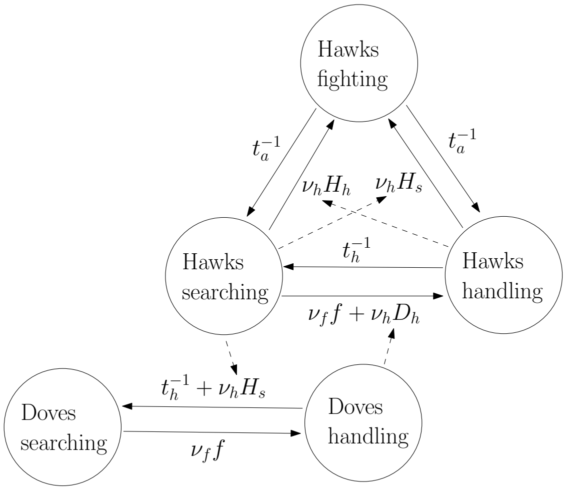

FIGURE 1. Phase diagram for deterministic model of Hawk - Dove kleptoparasitic population. The dashed arrows represent how certain states influence a transition between other states.

of a food is the time needed to find it, i.e. $V = (\nu_f f)^{-1}$. The cost of the fight is the time both Hawks spend fighting. Since each of them spends time $\frac{t_a}{2}$, we have $\frac{C}{2} = \frac{t_a}{2}$.

There are two conclusions - qualitative and quantitative - derived in [6] about this model; and both conclusions are in an agreement with the abstract Hawk-Dove game studied in Section 2. The qualitative conclusion is that the Hawk-only population is stable if and only if $C < V$, i.e. if

$$t_a < (\nu_f f)^{-1}$$

This is equivalent to saying that the food is, on average, acquired faster by the aggressive fighting than by a non-aggressive searching.

If $C > V$, then searching takes less time than winning the fight and, consequently, a population of all Hawks can be invaded by Doves. Also, Hawks can invade Doves under any circumstance and thus the population tends to a stable equilibrium where both Hawks and Doves coexist. The quantitative conclusion is that the fraction of Hawks in the stable mixture is given by

$$\frac{1}{\nu_f f t_a} = \frac{V}{C}$$

Notice that both conclusions are completely independent of the density of the population, the mean handling time, and the rate for searching handlers.

4. STOCHASTIC MODEL OF KLEPTOPARASITIC HAWK-DOVE GAME

The following stochastic model extends the model studied in [32] in the spirit of the deterministic model introduced in Section 3. We have two types of individuals, Hawks and Doves, behaving as described above. Although the schematic description of the model is the same as for the deterministic model and as given by Figure 1, the important distinction is that the stochastic model deals with the exact count of

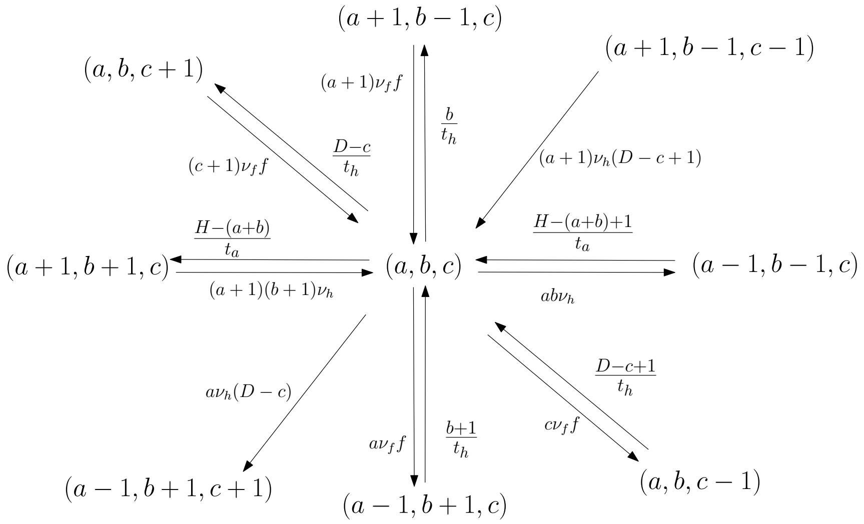

FIGURE 2. Transition diagram for a general state (a, b, c) - a state of the population where currently a Hawks are searching, b Hawks are handling, $H-(a+b)$ Hawks are fighting, c Doves is searching and $D-c$ Doves is handling. The numbers by the arrows represent the transition rates. Going south represents a Hawk found a food item, going north represents a Hawk finished handling, going east means two Hawks started fight, going west means Hawks finished a fight, going southwest means a Hawk found a handling Dove, going southeast means a Dove found a food item, going northwest means a Dove finished handling.

the individuals rather than with densities. Specifically, the dynamics in the finite population can be described by a Markov chain. At any given point of time, the state of the population consisting of H Hawks and D Doves in total can be described by a triple of (H_s, H_h, D_s), representing the number of searching Hawks, handling Hawks, and searching Doves. Clearly, the number of handling Doves equals $D - D_s$ and the number of fighting Hawks equals to $H-(H_s+H_h)$. Since the fights are between pairs of individuals only, $H-(H_s+H_h)$ must be even. Hence the states of the population can be identified with

$$\{(H_s, H_h, D_s), 0 \le H_s, H_h \le H, 0 \le D_s \le D, H-(H_s+H_h) \text{ is even}\}$$

For a fixed D_s, there are in total on the order of $H^2/4$ states, [32]. Thus, we have on the order of

$$H^2(D+1)/4$$

states of the dynamics. The transitions between those states are given in the Figure 2.

The scheme on Figure 2 can eventually lead to the system of roughly $H^2(D+1)/4$ linear equations describing the equilibria of the population. The system could theoretically be solved using software packages like Maple (see [12] where similar calculations are being done) and we could thus calculate a measure of fitness -

a proportion of the time a single Hawk or a single Dove is spending by handling a food item.

Here we concentrate on the qualitative analysis - a question when a single Dove can invade a population of Hawks. Due to the stochastic nature of the model, this question cannot be solved by simply calculating the average amount of time a Hawk or a Dove spend by handling - even when the average is lower for a Dove, there can be a nonzero chance that Dove will actually do better than Hawks. Indeed, assuming a hypothetic scenario where the proportion of time Doves spend handling is uniformly distributed in $[0, 0.5]$ (thus averaging 0.25), while the proportion of time Hawks spend handling is uniformly distributed in $[0, 0.75]$ (thus averaging 0.375). Yet, there is only a 0.625 chance Hawks spend more time handling than Doves in any particular realization.

Thus, we try to solve the following problems

1) What is a chance that in a population of N Hawks and a single Dove, Hawks do better?
2) How does the chance depend on the Hawks count N and parameter values $\nu_f f, t_a, t_h, \nu_h$?

5. Methods

Instead of solving the (potentially huge) system of linear equations based on Figure 2, we perform the Markov Chain Monte Carlo simulations. We look at the population as a system of chemical substances where the following reactions take place:

$$\begin{aligned}
\text{searching Hawk} &\xrightarrow{\nu_f f} \text{handling Hawk}\\
\text{searching Dove} &\xrightarrow{\nu_f f} \text{handling Dove}\\
\text{handling Hawk} &\xrightarrow{t_h^{-1}} \text{searching Hawk}\\
\text{handling Dove} &\xrightarrow{t_h^{-1}} \text{searching Dove}\\
\text{searching Hawk} + \text{handling Dove} &\xrightarrow{\nu_h} \text{handling Hawk} + \text{searching Dove}\\
\text{searching Hawk} + \text{handling Hawk} &\xrightarrow{\nu_h} \text{pair of fighting Hawks}\\
\text{pair of fighting Hawks} &\xrightarrow{2/t_a} \text{searching Hawk} + \text{handling Hawk}
\end{aligned}$$

We implement Gillespie's algorithm as described in [14]. The algorithm assumes that at every point of time, there is at most one reaction happening (this is in agreement with the scheme in Figure 2). For an input, it takes the number of reactants, the speeds of the equations, and the inner time of the system. Given this input, it generates

a) when the next reaction occurs;
b) what reaction it will be.

Based on the above, it updates the number of reactants and the inner time of the system and the cycle repeats itself. The algorithm stops when either enough reactions were performed or when the inner time is greater than predetermined value T_{max}. We tracked the proportion of time an average Dove and Hawk spend in a handling state (to assure compatibility with the description of the Markov chain at Figure 2 and with the models introduced in [23, 32]). We set $T_{max} = 10$

in all of the reported simulations as this proved to be the threshold when outcomes did not depend significantly on T_{max} anymore. For each combination of N and parameters $\nu_f f, t_a, t_h, \nu_h$ we ran 10,000 simulations and tracked the proportion π when an average Hawk did better than Dove.

6. Results

In this section we describe and discuss how π depends on N, $\nu_f f, t_a, t_h$, and ν_h.

6.1. **Dependence on $\nu_f f t_a$ and the comparison with deterministic model.** Both the stochastic and the deterministic models agree on the importance of a relationship between $(\nu_f f)^{-1}$ and t_a. The first is an expected time to acquire food by searching, the other is an expected time to acquire food by fighting. No matter what model is assumed, if $\nu_f f t_a < 1$, i.e. if time to get food by fighting is shorter than the time to get food by searching, Hawks do better overall, and if $\nu_f f t_a > 1$, Hawks generally do worse overall.

The difference between the stochastic and deterministic models is that, under the deterministic model, $\nu_f f t_a = 1$ is a very strict threshold and Hawks either always do better or always do worse than Doves, depending on what side of the threshold the value of $\nu_f f t_a$ is. Also, in the deterministic model, the outcome does not depend on any other parameter value, including the density of the individuals, time to eat food, or speed to find a handler.

In the stochastic model, on the other hand, the outcome depends on all of the parameters. More importantly, Hawks are never 100% better than an invading Dove and the proportion of the time when Hawks are better continuously decreases from almost 1 (in most cases) to almost 0 as $\nu_f f t_a$ increases from 0 to ∞. See Figure 3.

Also, notice that in the stochastic model, Hawks do better even if $\nu_f f t_a > 1$ as long as $\nu_f f t_a$ is not too large. This effect is particularly strong for a population with a small number of Hawks or when fights are long (small value of $2/t_a$) - both circumstances yield to the condition when there is not enough handling Hawks (either because there is not enough Hawks in total, or because the Hawks that are in the population are busy fighting). And when there is only a small number of handling Hawks, a handling Dove (as one of the very few handlers) is under greater danger of being found by a searching Hawk.

6.2. **Dependence on N.** The proportion π depends on the size of Hawk population, N. The exact relationship depends on the value of $\nu_f f t_a$ being smaller than, equal to, or greater than 1, see Figure 4.

If $\nu_f f t_a < 1$, then the proportion π increases as N increases (for small N) and then the proportion becomes stable for $N \approx 20$. If $\nu_f f t_a = 1$, the strategy of Hawks is better in 58.85 % cases on average (std. deviation 0.59 %) and the result is evidently independent of the number of Hawks N. If $\nu_f f t_a > 1$, the proportion when the strategy of Hawks is better decreases with increasing small values of N, but the proportion becomes stable for about $N = 20$.

The reason why outcomes stabilize is that although we measure π, we are practically measuring what the impact is and how big the impact is with the different parameter values on a single Dove that entered a population of Hawks. Since a Dove then represents roughly $1/(N+1)$ of the population, the impact gets negligible as N increases.

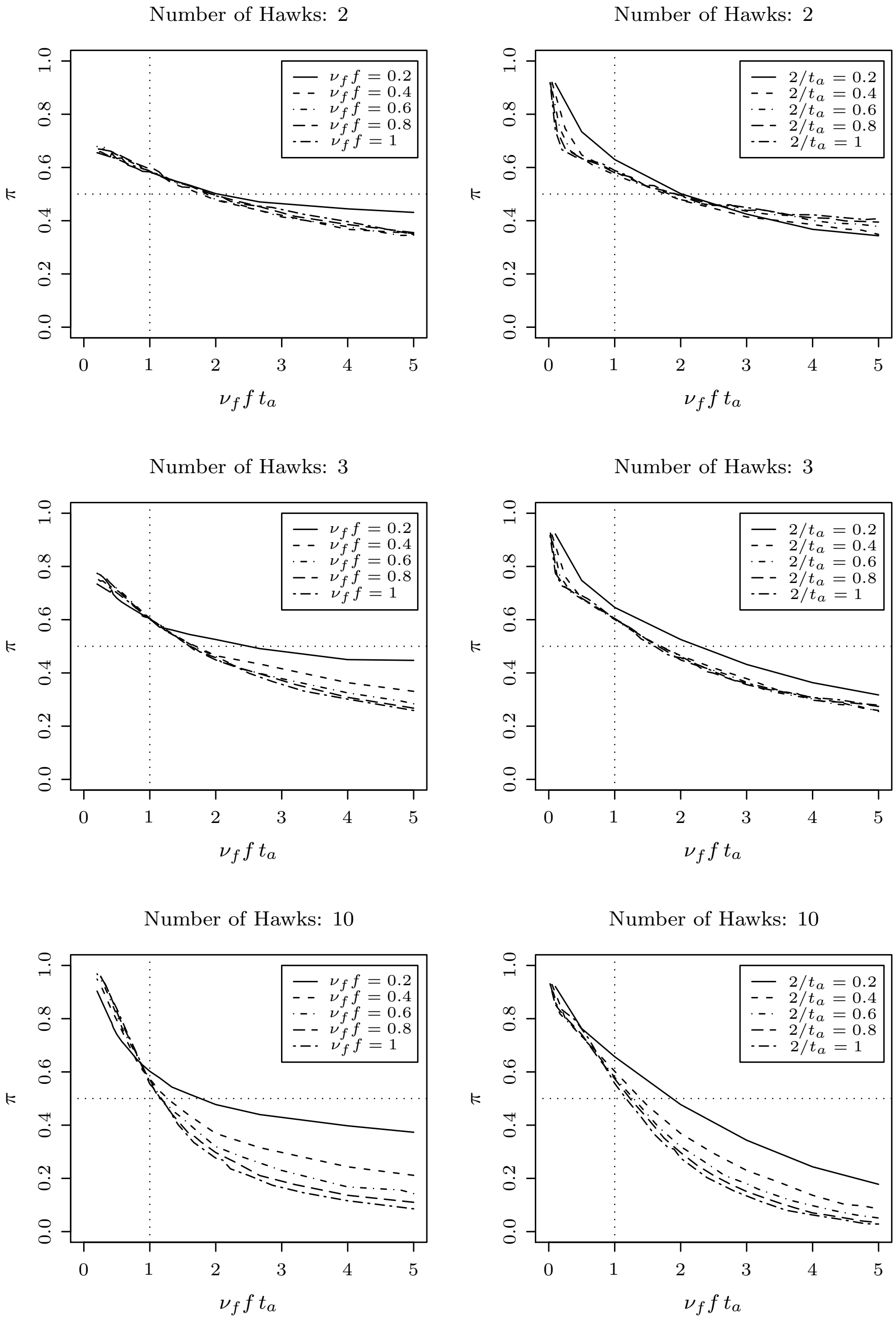

FIGURE 3. Dependence of π on $\nu_f f t_a$. In all cases $\nu_h = 0.7, 1/t_h = 0.7$

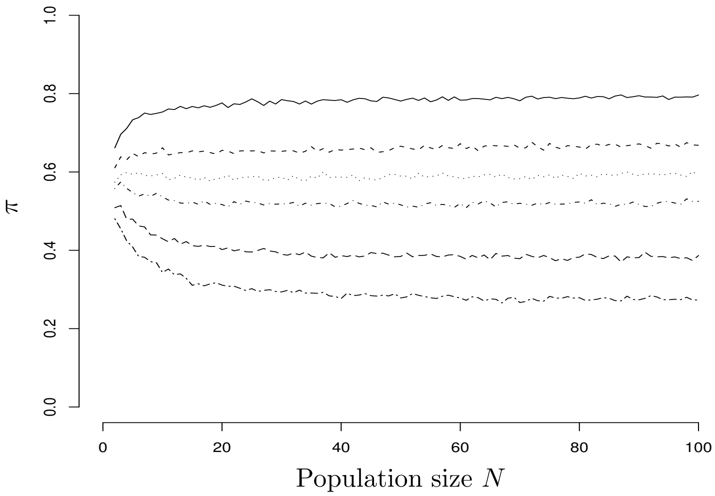

FIGURE 4. Dependence of π on N. (Legend: $\nu_f f t_a = 0.4$ solid, $\nu_f f t_a = 0.8$ dashed, $\nu_f f t_a = 1$ dotted, $\nu_f f t_a = 1.2$ dotdashed, $\nu_f f t_a = 1.6$ longdashed, $\nu_f f t_a = 2$ twodashed). In all cases $\nu_h = 0.7, 1/t_h = 0.7$.

TABLE 2. Statistics of the simulations for the dependence of π on N. In all cases $\nu_h = 0.7, 1/t_h = 0.7$.

$\nu_f f t_a < 1$ and $N = 2, \ldots, 100$							
$\nu_f f t_a$	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Std. Dev.
0.4	0.6615	0.7752	0.7846	0.7784	0.7901	0.7966	0.0205
0.8	0.6106	0.6532	0.6597	0.6586	0.6662	0.6761	0.0102
$\nu_f f t_a < 1$ and $N = 20, \ldots, 100$							
0.4	0.7645	0.7818	0.7869	0.7857	0.7907	0.7966	0.0064
0.8	0.6468	0.6553	0.6618	0.6616	0.6672	0.6761	0.0072
$\nu_f f t_a = 1$ and $N = 2, \ldots, 100$							
1	0.5741	0.5847	0.5887	0.5885	0.5926	0.6007	0.0059
$\nu_f f t_a > 1$ and $N = 2, \ldots, 100$							
1.2	0.5086	0.5170	0.5196	0.5216	0.5234	0.5746	0.0104
1.6	0.3701	0.3831	0.3871	0.3966	0.3972	0.5142	0.0266
2.0	0.2656	0.2774	0.2831	0.2974	0.2980	0.4805	0.0388
$\nu_f f t_a > 1$ and $N = 20, \ldots, 100$							
1.2	0.5086	0.5157	0.5187	0.5185	0.5215	0.5314	0.0047
1.6	0.3701	0.3815	0.3853	0.3866	0.3908	0.4062	0.0072
2.0	0.2656	0.2751	0.2811	0.2828	0.2895	0.3111	0.0100

The table 2 summarizes the statistics of the outcomes of the simulations.

The results are qualitatively in agreement with the deterministic model. If $\nu_f f t_a < 1$, i.e. if it is on average faster to acquire food by fighting than by searching, Hawks should do better; and the more Hawks, the smaller chances for a single Dove to finish handling undisturbed. The difference between the deterministic and stochastic models is that a) Hawks never do better in 100% of the cases; and b) after the Hawk population reaches a certain threshold, the chances of a Dove doing better stabilizes (at relatively high nonzero value).

6.3. Dependence on $\nu_f f$.

The proportion π decreases as $\nu_f f$ increases. Moreover, the rate at which it decreases is dependent on $2/t_a$ and N - with the greater t_a and the greater N, the proportion π decreases faster. See Figure 5.

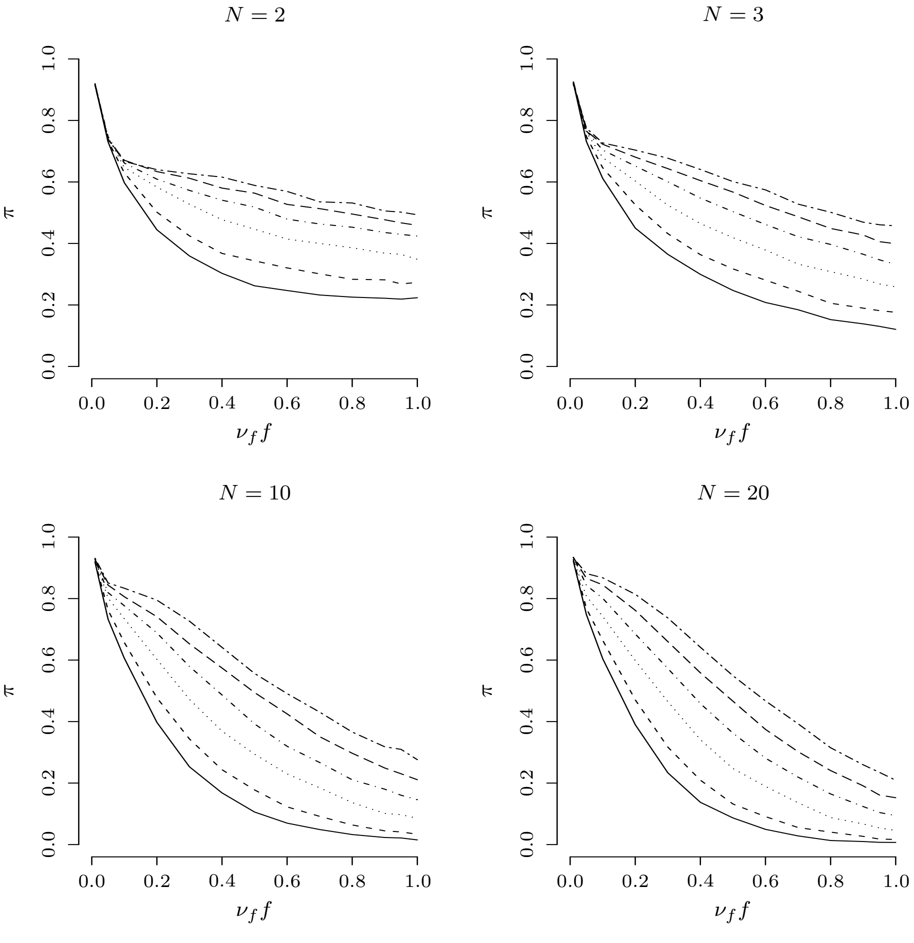

Figure 5. Dependence of π on $\nu_f f$ for fixed $\frac{t_a}{2}$. (Legend: $2/t_a = 0.1$ solid, $2/t_a = 0.2$ dashed, $2/t_a = 0.4$ dotted, $2/t_a = 0.6$ dotdashed, $2/t_a = 0.8$ longdashed, $2/t_a = 1.0$ twodashed). In all cases $\nu_h = 0.7, 1/t_h = 0.7$.

The first is a natural result since the more food is available in the population, the more likely it is for Hawks to find food rather than a handling Dove. Consequently, a chance of a Dove doing better than Hawks increases as the density of food increases. We did not perform extensive simulations for too large values of $\nu_f f$; however it is clear that in a limiting case of $\nu_f f \approx \infty$, there would be no distinction between Hawks and Doves since any individual would find food almost instantly and thus we have that $\pi \to 0.5$ as $\nu_f f \to \infty$. Consequently, π as a function of $\nu_f f$ is eventually increasing.

The second result can be explained as the following. If the fights last longer and if there are more Hawks in the population, the more Hawks are engaged in fights rather than in handling, i.e. the initial decrease of π is steeper with larger t_a and larger N.

6.4. **Dependence on t_a.** It can be seen from Figure 6 that π increases as t_a decreases.

This is natural, since the shorter the fights are, the better the Hawks do. The effect of t_a is actually doubled, because the less time Hawks spend fighting, the more time they spend handling (which increases their fitness) but also by searching (which decreases the fitness of a Dove since Hawks are looking for food items as well as handlers).

Further, the proportion π is higher with smaller values of $\nu_f f$ and the same value of t_a. This is because the chance of a Dove being found while handling increases if there is not enough food to search for. Moreover, the influence of $\nu_f f$ on π is stronger for larger N.

6.5. **Dependence on ν_h.** The dependence of π on ν_h is demonstrated on Figure 7. The value of ν_h influences π in conjunction with $\nu_f f t_a$ and N. The influence of ν_h on π is more profound for smaller N and essentially vanishes as N gets larger (reading the Figure 7 from top down). Also, the behavior of π as a function of ν_h changes from increasing (when $\nu_f f t_a < 1$), through constant (if $\nu_f f t_a = 1$), to decreasing (if $\nu_f f t_a > 1$). Further, we observe that points are systematically spread according the value of the parameter t_h. If $\nu_f f t_a < 1$, then π is a bit larger with the greater value of t_h and the same value of ν_h. If $\nu_f f t_a > 1$, then π is a bit smaller with the greater value of t_h and the same value of ν_h.

All of these observations are again natural. For large N, chances of a handling Dove being found by a Hawk does not depend too much on ν_h - there are already enough Hawks to look for a handler, and the handler itself can be either a Hawk or a Dove - hence a relatively small change of ν_h will not make a significant difference for a Dove.

Increasing ν_h results in all handlers, Hawks and Doves, being found faster. Thus, the direction of dependence of π on ν_h is influenced by the value $\nu_f f t_a$. When $\nu_f f t_a < 1$, i.e. when it is better to fight than to look for food, increasing ν_h improves the fitness of Hawks since a Dove is being found faster (and thus cannot eat so much food). Hawks are found faster as well; but the fights are relatively short and so the harm to Hawks is smaller than the harm to a Dove. On the other hand, if $\nu_f f t_a > 1$, it is better to look for food than fight and thus increasing ν_h decreases π. It is true that a handling Dove is being found more often as ν_h increases, but handling Hawks are found more often as well and, since fighting is not beneficial, the harm to Hawks is greater.

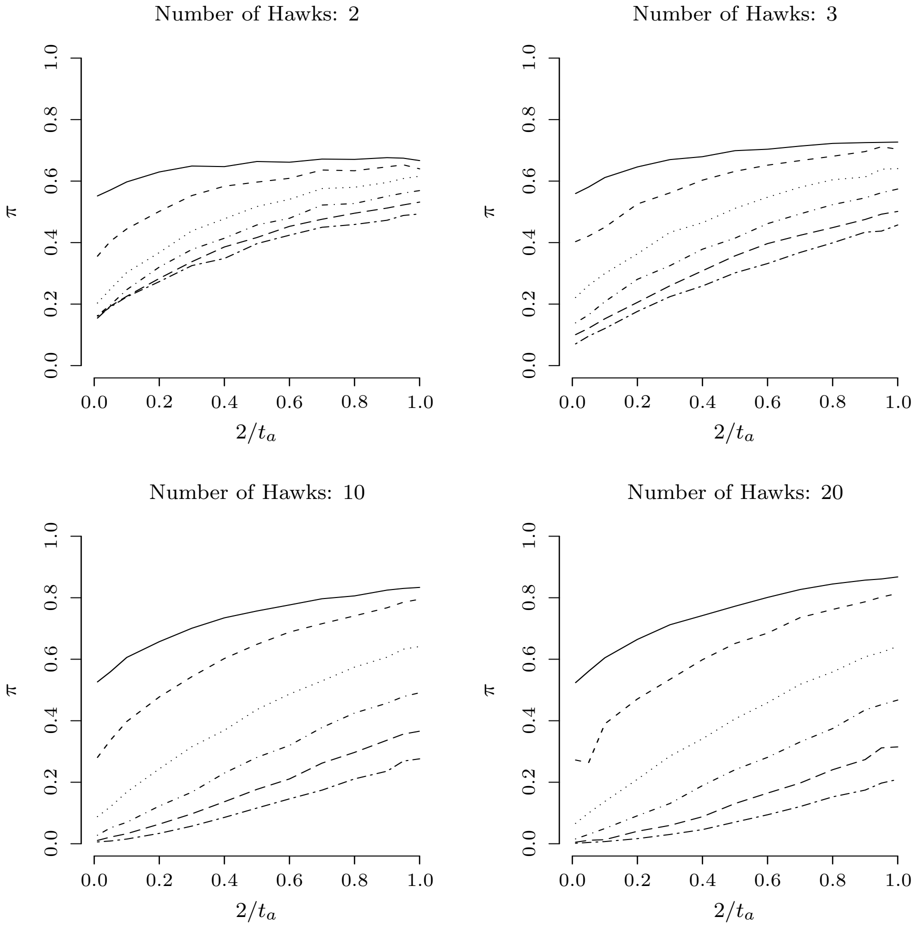

FIGURE 6. Dependence ESS on $2/ta$ for fixed $\nu_f f$. (Legend: $\nu_f f = 0.1$ solid, $\nu_f f = 0.2$ dashed, $\nu_f f = 0.4$ dotted, $\nu_f f = 0.6$ dotdashed, $\nu_f f = 0.8$ longdashed, $\nu_f f = 1.0$ twodashed). In all cases $\nu_h = 0.7, 1/t_h = 0.7$

The effect of $1/t_h$ on the dependence of π on ν_h can be explained by a similar argument. As $1/t_h$ increases, individuals spend less and less time handling and thus there is a smaller chance for a searcher to encounter a handler. If $\nu_f f t_a < 1$ (Hawks do better if they fight), the above benefits a Dove contrary to the effect on increasing ν_h which benefits Hawks. If $\nu_f f t_a > 1$ (Hawks do better if they do not fight), the increasing $1/t_h$ benefits Hawks contrary to the effect on increasing ν_h which benefits a Dove. In other words, in the case of $\nu_f f t_a < 1$, the value of proportion π is declined, and, in the case of $\nu_f f t_a > 1$, the value of proportion π is advanced by the impact of increasing $1/t_h$.

6.6. **Dependence on t_h.** Figure 8 shows the relation between π and $1/t_h$. The value of t_h influences π in conjunction with $\nu_f f t_a$ and N. Similarly to the dependence of π on ν_h, the influence of t_h on π is more profound for smaller N and

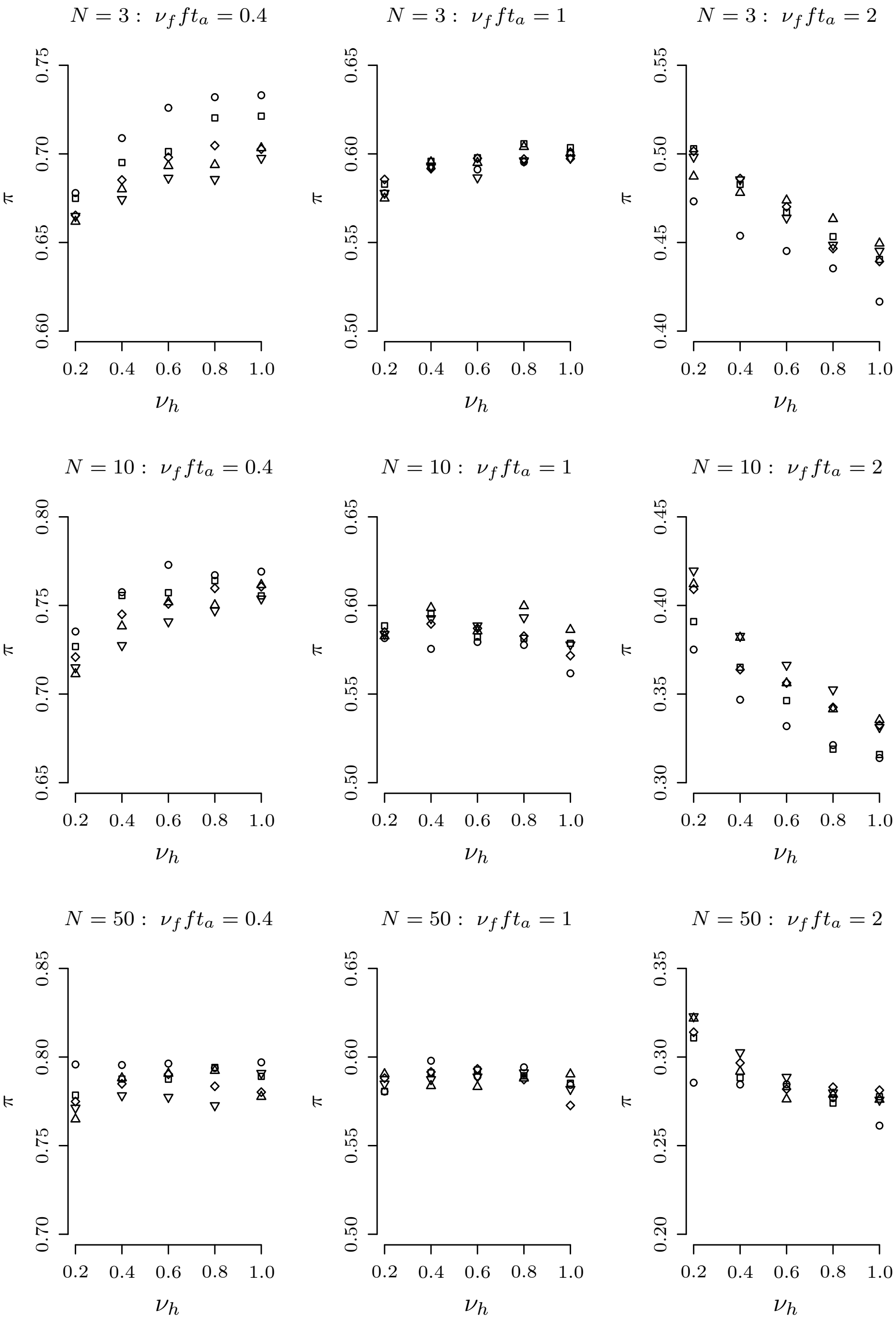

FIGURE 7. Dependence ESS on ν_h for fixed t_h. (Legend: ◯ $1/t_h = 0.2$, □ $1/t_h = 0.4$, ◇ $1/t_h = 0.6$, △ $1/t_h = 0.8$, ▽ $1/t_h = 1.0$.)

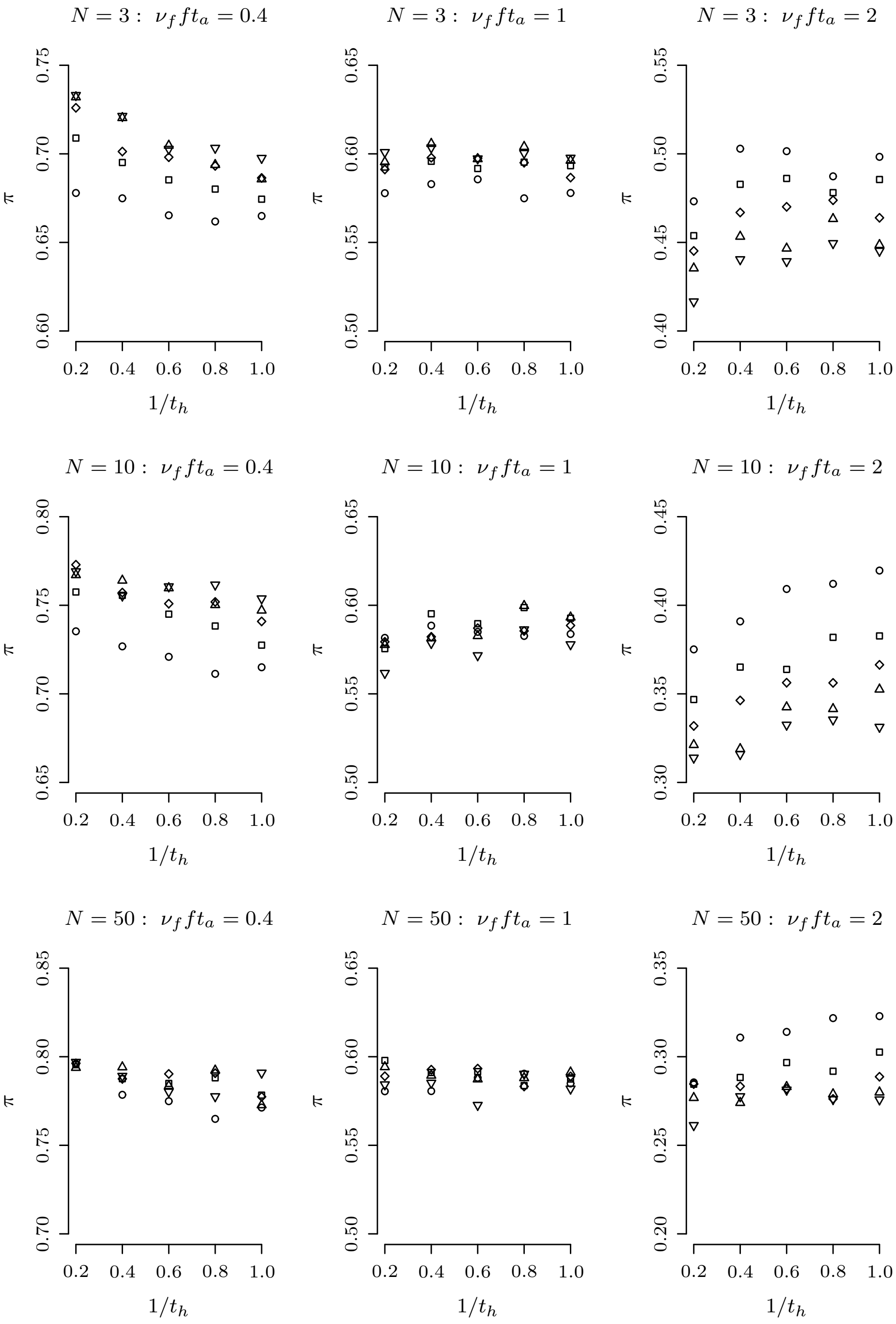

FIGURE 8. Dependence of π on t_h for fixed ν_h. (Legend: ◯ $\nu_h = 0.2$, □ $\nu_h = 0.4$, ◊ $\nu_h = 0.6$, △ $\nu_h = 0.8$, ▽ $\nu_h = 1.0$.)

almost vanishes as N gets larger (reading the Figure 8 from top to bottom). Also, the behavior of π as a function of $1/t_h$ changes from decreasing (when $\nu_f f t_a < 1$), through constant (if $\nu_f f t_a = 1$) to increasing (if $\nu_f f t_a > 1$). Further, we observe that points are systematically spread according the value of the parameter ν_h. If $\nu_f f t_a < 1$, then π is a bit larger with the greater value of ν_h and the same value of t_h. If $\nu_f f t_a > 1$, then π is a bit smaller with the greater value of ν_h and the same value of t_h.

These results can be explained in the same way as in previous a case. By comparing corresponding graphs in Figure 7 and 8 we see that the relationship between π and ν_h is stronger than between π and $1/t_h$.

7. Summary

In this paper we have extended the classical Hawk-Dove game to model the finite population of kleptoparasitic individuals.

We have demonstrated that there are some similarities between the introduced stochastic description and the known deterministic model. In both cases, the major indicator of the success or failure of Hawk strategy was a value of $\nu_f f t_a$. When fights are beneficial (if $\nu_f f t_a < 1$, i.e. if food is rare or fights are short) Hawks do better; when fights are not beneficial, Hawks do worse. This rule was a sole indicator for Hawk's success in the deterministic model of infinite populations, [6]. On the other hand, the situation in finite population was much richer and the exact dependence was determined by a variety of natural parameters. In fact, any parameter of our model (either by itself or in conjunction with other parameters) had an impact on the outcome. Since we have naturally explained the impact of any of the parameters, we argue that the stochastic model is much closer to the description of the reality than a deterministic one.

Further analysis needs to be done. Recently, evolutionary game dynamics as well as a notion of evolutionary stable strategy in finite populations was introduced in [31] and [24]. We hope to apply the methods and results from those papers to our particular model and to investigate the Hawk-Dove game in finite kleptoparasitic population from that point of view.

Also we would like to point out that, to assure compatibility with deterministic as well as recently introduced stochastic models ([23, 32]), we measured a fitness by measuring a fraction of the time the individuals spent by handling. It is proved [7] that, for infinite populations and independently of the individual type, this fraction is proportional to the number of food items eaten. Our simulation allows us to count the number of food items directly and we are currently investigating whether the same relationship holds for finite populations as well.

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