PALMETTO NUMBER THEORY SERIES (PANTS) XXVI

All talks are in Petty 219.

Saturday, September 17, 2016

9:30–10:00: Coffee outside Petty 219

10:00–11:00: Krishnaswami Alladi (University of Florida)
The distribution of the number of prime factors with restrictions—variations of the classical theme

The study of $\nu(n)$ the number of prime factors of n began with Hardy and Ramanujan in 1917 who showed that $\nu(n)$ has normal order $\log\log n$ regardless of whether the prime factors are counted singly or with multiplicity. Their ingenious proof of this utilized uniform upper bounds for $N_k(x)$, the number of integers up to x with $\nu(n) = k$. Two major results followed a few decades later—the Erdös–Kac theorem on the distribution more generally of additive functions, and the Sathe–Selberg theorems on the asymptotic behavior of $N_k(x)$ as k varies with x—a significant improvement of Landau's asymptotic estimate for $N_k(x)$ for fixed k. We shall consider the distribution of the number of prime factors by imposing certain restrictions—such as (i) requiring all prime prime factors of n to be $< y$ (the important case of smooth numbers), and (ii) considering only the prime factors $< y$, but for all integers. For (i), I showed in 1982 how an interesting variation of the classical theme with regard to the variance of $\nu(n)$ takes place when $\log x/\log y$ is large, and this led to further work by Hildebrand, Tenenbaum, Hensley and myself on the Erdös–Kac Theorem for smooth numbers. Very recently, I noticed a surprising variation of the classical theme in the case (ii) with regard to the *local distribution*. Details of the asymptotic analysis of the local distribution in (ii) with emphasis on uniformity in y is being carried out now by my PhD student Todd Molnar. Our approach involves the interplay of a variety of methods such as the Perron integral formula, the Hankel contour for the Gamma function, Selberg's method, Buchstab iteration, and difference-differential equations to achieve uniformity. Tenenbaum has indicated recently in communication that by a careful analysis involving the Selberg–Delange method, the error terms can be improved in certain crucial ranges.

11:15–11:35: Luke Giberson (Clemson University)
Champion Primes of Rational Elliptic Curves on Average

For a rational elliptic curve E, we say a prime of good reduction p is a champion prime of E if the group of $\mathbb{F}_p$-rational points is as large as possible (according to the Hasse bound). Upon averaging over a family of elliptic curves, we show an asymptotic result on the number of champion primes less than a real parameter X. This work is joint with Kevin James.

11:45–12:05: Saikat Biswas (Arizona State University)
Capitulation, unit groups, and the cohomology of S-idele classes
Consider a finite cyclic extension L/K of number fields and the set S of all infinite primes of L. In this talk, we relate the S-idele class group of L to the capitulation map of L/K as well as to the cohomology of the unit group of L.

12:15–2:00: Lunch break

2:00–3:00: David Roe (University of Pittsburgh)
Algebraic tori and counting p-adic fields
Algebraic tori play a central role in the structure theory and representation theory of algebraic groups. I will describe an ongoing project to investigate algebraic tori over p-adic fields. The project naturally divides into two parts: finding finite subgroups of $\mathrm{GL}_n(\mathbb{Z})$ and listing all p-adic fields with a given Galois group. On the $\mathrm{GL}_n(\mathbb{Z})$ side, I will describe prior work that computes these subgroups for small n. On the Galois side, I will explain how to count the number of extensions with a given Galois group, and give an algorithm for finding such fields.

3:15–3:35: Jonathan Milstead (UNCG)
Computing Galois Groups of Eisenstein Polynomials over p-adic Fields
An exploration of methods for computing Galois groups of Eisenstein polynomials that combine the ramification polygon approach and resolvent methods.

3:45–4:30: Nicolas Simard (McGill University)
Petersson Inner Products of Binary Theta Series
Modular forms are a fascinating subject. On the one hand, spaces of modular forms are relatively simple to study and are "computer-friendly". On the other hand, modular forms (or their L-functions) contain a lot of arithmetic information about interesting geometric objects (think of the Birch and Swinnerton-Dyer conjecture, for example). To an imaginary quadratic field K, one can naturally attach a collection of theta series. A question one might ask is the following: do the Petersson inner products of these theta series contain arithmetic information about K? In this talk, we make a first step in answering this general question by giving explicit formulas for the Petersson inner products of those theta series and an algorithm to compute them. We also present some numerical examples.

Sunday, September 18, 2016

9:00–9:30: Coffee outside Petty 219

9:30–10:30: Roger Baker (Brigham Young University)
Recent progress in Weyl sums and their applications
This year is the 100th anniversary of the introduction of *Weyl sums*, that is, exponential sums with polynomial argument $f(x)$, in a paper by Weyl on uniform

distribution modulo one. I discuss some of the significant events in our understanding of Weyl sums since then, and recent progress that emerges as a consequence of the proof of the main conjecture for Vinogradov's mean value (Wooley and Bourgain–Demeter–Guth), For example we now know more about inequalities for $f(p)$ modulo one, where p runs over the primes, in the case where f has irrational leading coefficient.

10:45–11:05: Stevo Bozinovski (South Carolina State University)
A property of Riemann zeta function

The following result is obtained: Theorem: $\zeta(-1) = \eta(-2)$ if their corresponding Dirichlet series have same odd number of elements. This work is a collaboration with Adrijan Bozinovski.

11:15–11:35: Abbey Bourdon (University of Georgia)
Torsion in Isogeny Classes of CM Elliptic Curves

Let F be a number field and let $E_{/F}$ be an elliptic curve with complex multiplication (CM). That is, we assume the ring of endomorphisms of E defined over the algebraic closure of F is isomorphic to an order $\mathcal{O}$ in an imaginary quadratic field K. We wish to understand the structure of the group of rational torsion points of E, denoted $E(F)[\text{tors}]$, in the case where F contains K. If $\mathcal{O} = \mathcal{O}_K$, the full ring of integers in K, one may avoid the technical complications associated with the study of non-maximal orders, so it is desirable to understand to what extent one may "reduce to the maximal order." In this talk, we will show that $\#E(F)[\text{tors}]$ is bounded by $\#E'(F)[\text{tors}]$, where $E'_{/F}$ is an $\mathcal{O}_K$-CM elliptic curve isogenous to E. This is joint work with Pete L. Clark.

11:45–12:05: Jeremy Rouse (Wake Forest University)
The density of primes dividing a term in the Somos-5 sequence

The Somos-5 sequence is defined by $a_0 = a_1 = a_2 = a_3 = a_4 = 1$ and $a_m = \frac{a_{m-1}a_{m-4}+a_{m-2}a_{m-3}}{a_{m-5}}$ for $m \geq 5$. We relate the arithmetic of the Somos-5 sequence to the elliptic curve $E : y^2+xy = x^3+x^2-2x$ and use properties of Galois representations attached to E to prove the density of primes p dividing some term in the Somos-5 sequence is equal to $\frac{5087}{10752}$.