



Summer School in Computational Number Theory

May 22 to May 26, 2017

Modular Forms

Speakers

- Matt Greenberg (University of Calgary)
- Paul Gunnells (UMass Amherst)
- Mark McConnell (Princeton University)
- David Roe (University of Pittsburgh)



organized by the number theory group at UNCG

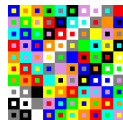
www.uncg.edu/numbertheory/summerschool

Introduction and research interests

Angelica Babei

Dartmouth College

May 22nd, 2017



Tiled orders in $M_n(k)$

Let k be a non-archimedean local field, \mathcal{O} its valuation ring, and $\Gamma \subset M_n(k)$ an order (a subring containing 1, finitely generated over \mathcal{O} such that $k \otimes_{\mathcal{O}} \Gamma = M_n(k)$).

TFAE:

1) Γ is tiled (graduated/split)

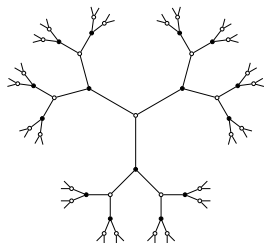
2) Γ contains a conjugate of the ring $R = \begin{pmatrix} \mathcal{O} & & & \\ & \mathcal{O} & & \\ & & \ddots & \\ & & & \mathcal{O} \end{pmatrix}$.

3) Γ can be written as the intersection of n maximal orders, uniquely determined by Γ .

Finding the normalizer $\mathcal{N}(\Gamma) \subset M_n(k)$

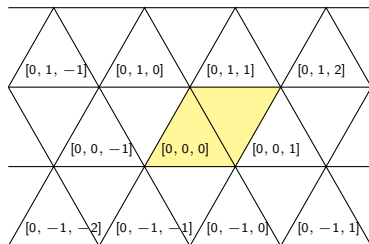
$n = 2$ (Hijikata): For the Hecke trace formula, he used that if Γ is not maximal, then $\mathcal{N}(\Gamma)/k^\times \Gamma^\times \cong \mathbb{Z}/2\mathbb{Z}$. We can also get this from the action of $GL_2(k)$ on the Bruhat-Tits tree of $GL_2(k)$.

The tree for $SL_2(\mathbb{Q}_2)$:



$n \geq 3$: We can visualize the tiled orders as convex polytopes in apartments of the building for $SL_n(k)$. Then finding the normalizer amounts to finding symmetries of the convex polytopes in the apartment.

An example of such a polytope:



Research Interests

Matthew Bates

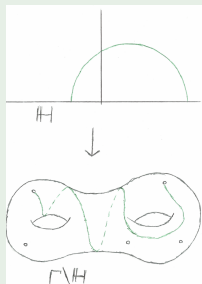
University of Massachusetts Amherst

UNCG Summer School in Computational Number Theory, 2017

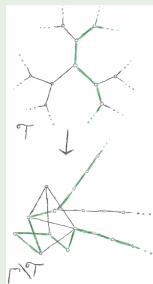
Bruhat-Tits Buildings (p -adic symmetric spaces)

- Given a Lie group G and a maximal compact subgroup K , the quotient space $X \stackrel{\text{def}}{=} G/K$ is a contractible Riemannian symmetric space with a natural G -action. If $\Gamma \subset G$ is a countable discrete subgroup of G , then the quotient $\Gamma \backslash X$ is a manifold. One can learn about Γ via studying the geometry of the quotient $\Gamma \backslash X$. For example $H^*(\Gamma, M) = H^*(\Gamma \backslash X, M)$ (if Γ is torsion free).
- Bruhat-Tits buildings provide a p -adic analogue of Riemannian symmetric spaces.

Example (SL_2)



$$\mathbb{H} = SL_2(\mathbb{R})/SO(2), \\ \Gamma \subseteq SL_2(\mathbb{Z})$$



$$\mathcal{T} \text{ comes from } SL_2(\mathbb{F}_2((1/t))), \\ \Gamma \subseteq SL_2(\mathbb{F}_2[t])$$

Questions/things I am thinking about

- What does the boundary look like?
- How can we (partially) compactify these spaces?
- What do the quotients look like? e.g. number of cusps, cohomology, etc...
- What do fundamental domains look like?
- How to develop a theory of modular symbols for $SL_3(\mathbb{F}_q((1/t)))$.
- How to calculate Hecke eigenvalues.

Ben Breen, Dartmouth College

3rd year graduate student

Research Interests:

Algebraic number theory and Arithmetic
Statistics

Old works:

*Wild ramification in a family of
low-degree extensions arising from
iteration* with Rafe Jones, Tommy
Occhipiniti, and Michelle Yuen



Research Projects

Heuristics on Narrow class group:

Developing Cohen Lenstra type heuristics for narrow class groups. Modeling unit groups and determining when fields have a system of totally positive units.

Hilbert Modular Forms:

Developing algorithms for multiplication of Hilbert Modular forms. Hopefully will be able to implement this in Sage or Magma

Introduction

Benjamin Carrillo

Arizona State University
School of Mathematical and Statistical Sciences

May 22, 2017



Current Research

In continuing the work of my advisor Prof. John Jones, my current research is calculating information about the ramification groups of various degree extensions of \mathbb{Q}_p . This information is finding the inertia subgroup and the Galois slope content of the extension.

How do we calculate this information?

We change the problem to a number field search.

For each p -adic extension K , we find a number field F where the p -adic completion is isomorphic to K and $\text{Gal}(K/\mathbb{Q}_p) \cong \text{Gal}(F/\mathbb{Q})$. In other words, find a number field F where there is only one prime above p over the splitting field of F . We can then apply algorithms and procedures to the number field, namely resolvents, and this will give us information about the ramification groups of the p -adic extension.



Introduction and Research Interests

Dartmouth College

Sara Chari

May 22, 2017

Quaternion Algebras

Let $B = \left(\frac{a,b}{\mathbb{Q}}\right) = \{t + xi + yj + zij : t, x, y, z \in \mathbb{Q}\} / (i^2 - a, j^2 - b, ij + ji)$.
Define a reduced norm $\text{nrd}: B \rightarrow \mathbb{R}$ by

$$\text{nrd}(t + xi + yj + zij) = t^2 - ax^2 - by^2 + abz^2.$$

Let \mathcal{O} be a maximal order in B . An element $\alpha \in \mathcal{O}$ is irreducible if $\text{nrd}(\alpha) = p$ is prime in \mathbb{Z} . We also have a reduced trace $\text{trd}: B \rightarrow \mathbb{R}$ by

$$\text{trd}(t + xi + yj + zij) = 2t.$$

The Permutation

There are finitely many primes of reduced norm p , so, Q induces a permutation τ_Q on the primes of norm p as follows:

$$\tau_Q(P_i) = P_j \quad \text{if} \quad P_i Q = Q' P_j$$

for some $Q' \in \mathcal{O}$ with $\text{nrd}(Q') = q$. The permutation is well-defined by “unique” factorization.

I am generalizing results about the sign of the permutation and cycle structures known for $B = \left(\frac{-1, -1}{\mathbb{Q}}\right)$ to maximal orders in other division quaternion algebras.

My Research Interests

Mariagiulia De Maria

University of Luxembourg

Université de Lille 1

May 22, 2017

What I have done so far...

Let $p \geq 3$ be a prime number and \mathcal{O} the valuation ring in a finite extension K of \mathbb{Q}_p . Let ϖ be a uniformizer and $k = \mathcal{O}/\varpi$ the residue field. Let

$$\bar{\rho} : G_{\mathbb{Q}} \longrightarrow \mathrm{GL}_2(k)$$

be a continuous representation that is unramified at p .

Theorem

If $\rho : G_{\mathbb{Q}} \longrightarrow \mathrm{GL}_2(\mathcal{O}/\varpi^m\mathcal{O})$ is a minimal deformation of $\bar{\rho}$, then ρ is modular of weight 1.

This is an application of an $R = T$ theorem of Calegari and Geraghty in their article *Modular Lifting beyond the Taylor-Wiles Methods*.

... and what I want to do next.

The next steps are the following:

- Can I apply the methods of Calegari and Geraghty to the case of partial weight 1 Hilbert modular forms over p -adic rings? And over rings modulo p^m ?
- Compute examples of such forms and of their associated Galois representations.

Introduction and Research Interests

Marcus Elia

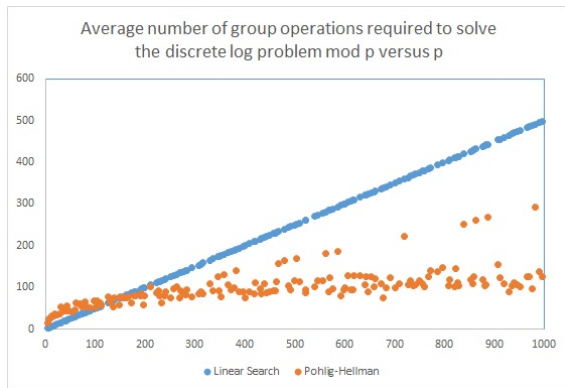
University of Vermont

UNCG Summer School

May 22, 2017

Research Interests

- Next Semester: Exploring Post-Quantum Cryptography
- Currently: Studying various algorithms to break the discrete logarithm problem.



Introduction

Nathan Fontes

Number Theory Summer School 2017 - UNCG

22 May 2017

Introduction

About

- Current second year Master's student at UNCG
- B.S. in Mathematics from UNC-Chapel Hill
- Research Interests - algebra, number theory, topology, combinatorics, graph theory
- Master's thesis next year with Dr. Dan Yasaki in the topic of arithmetree
- Undergraduate Research with Dr. Linda Green at UNC-CH in the topic of Fullerenes

Introduction

Past Research

- Fullerenes are carbon molecules formed like polyhedra, containing exactly 12 pentagons and the rest hexagons.
- Generalized Fullerenes are all polyhedra that can be created using only two types of polygons, with a specific number of one polygon.

k	Face 1	Face 2	Description
3	Hexagon	Triangle	Contains 4 Triangles
3	Hexagon	Square	Contains 6 Squares
3	Hexagon	Pentagon	Contains 12 Pentagons
4	Square	Triangle	Contains 8 Triangles

- The final row is what we call "Fourines" since they have 4 faces connecting at each vertex.

Yu Fu

Department of Mathematics and Statistics
University of Massachusetts Amherst
May 21, 2017

Introduction

- I'm 1st year graduate student in mathematics
- Undergraduate degree:
 - major: Mathematics
 Mechanical Engineering
 - minor: Statistics

Research Interests

- commutative algebra & algebraic geometry (especially invariant theory)
- arithmetic geometry (elliptic curves)

Current Research Interests

Hugh Roberts Geller

Clemson University

May 22, 2017



Siegel modular forms, Automorphic forms



Siegel modular forms, Automorphic forms

- $\mathfrak{H}_n := \{X + iY \in \text{Mat}_n^{\text{sym}}(\mathbb{C}) : X, Y \in \text{Mat}_n(\mathbb{R}), Y > 0\}$
- $\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \Gamma^n := \text{Sp}_{2n}(\mathbb{Z}),$

$$\gamma \cdot Z = (AZ + B)(CZ + D)^{-1}.$$



Research Interests

Siegel modular forms, Automorphic forms

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- We write $f \in \mathfrak{M}_k(\Gamma^n)$ if $f : \mathfrak{H}_n \rightarrow \mathbb{C}$ is holomorphic and for all $\gamma \in \Gamma^n$,

$$f(\gamma \cdot Z) = \det(CZ + D)^k f(Z).$$



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- Fourier Expansion: $f(Z) = \sum_{0 \leq T \in \Lambda_n} a(T; f) e^{2\pi i \text{tr}(TZ)}$



Congruences of Siegel Modular forms:



Research Interests

Congruences of Siegel Modular forms:

- For $f, g \in \mathfrak{M}_k(\Gamma^n)$ we say $f \equiv g \pmod{\mathcal{I}}$ if $a(T; F) \equiv a(T; g) \pmod{\mathcal{I}}$ for all $T \in \Lambda_n$.

Theorem (T. Kikuta, S. Takemori '14)

There exists a finite set $S_n(K)$ of prime ideals in K depending on n such that the following holds: For a prime ideal \mathfrak{p} of \mathcal{O} not contained in $S_n(K)$ and a mod \mathfrak{p}^m cusp form $F \in \mathfrak{M}_k(\Gamma^n)_{\mathcal{O}_{\mathfrak{p}}}$ with $k > 2n$, there exists $G \in \mathfrak{S}_k(\Gamma^n)_{\mathcal{O}_{\mathfrak{p}}}$ such that $F \equiv G \pmod{\mathfrak{p}^m}$.



Research Interests

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- Trying to generalize the result for level.



Introduction and research interests (so far)

Who? Seoyoung Kim

From? Brown University advised by J. H. Silverman
Seoyoung_Kim@brown.edu

When? May 22, 2017

Sato-Tate conjecture & Nagao's conjecture

The Sato-Tate conjecture

Given an elliptic curve E over \mathbb{Q} or a number field, we can define its trace of Frobenius as

$a_p(E) = p + 1 - \#E(\mathbb{F}_p)$ for each prime p . The Sato-Tate conjecture is about the distribution of trace of Frobenius, more precisely, if one denotes $\theta_p = \cos^{-1}(a_p/2\sqrt{p})$,

$$\lim_{X \rightarrow \infty} \frac{\#\{p \leq X : \alpha \leq \theta_p \leq \beta\}}{\#\{p \leq X\}} = \frac{2}{\pi} \int_{\alpha}^{\beta} \sin^2 \theta d\theta.$$

There is a generalization of the Sato-Tate conjecture for the higher genus case via random matrix theory.

Nagao's conjecture

Interestingly, Nagao conjectured if

$$A_p(E) = \frac{1}{p} \sum_{t=0}^p a_p(E),$$

$$\lim_{X \rightarrow \infty} \sum_{p \leq X} -A_p(E) \log p = \text{rank } E(\mathbb{Q}(T)).$$

Elliptic Divisibility Sequence

I'm interested in the possible application to the Sato-Tate conjecture (or various conjectures regarding trace of Frobenius) on Nagao's conjecture.

Elliptic Divisibility Sequence (EDS)

I'm also fascinated by the *elliptic divisibility sequence* (EDS) which is a natural generalization of the Fibonacci sequence on a fixed elliptic curve. It is defined by iterating a point $P \in E(\mathbb{Q})$ and looking at the denominator of the minimal form

$$x([n]P) = \frac{A_n}{D_n^2},$$

and the sequence $\{D_n\}_{n \geq 1}$ is called EDS. I'm trying to find the analogues of the classical questions for the Fibonacci sequence from EDS. Also, there are interesting ways to bridge EDS problems to the distribution of trace of Frobenius.

Introduction and Research Interests

Andrew J. Kobin

`ak5ah@virginia.edu`



22 May, 2017

Research Interests

Research Interests

Research Interests

- Bachelor (Wake Forest University, '13, with Hugh Howards)
 - Knot mosaic theory
 - “Crossing Number Bounds in Knot Mosaics”, with Howards, to appear in *Knot Theory and Its Ramifications*
- Master's (Wake Forest University, '15, with Frank Moore)
 - Class field theory and quadratic forms
 - Primes of the form $p = x^2 + ny^2$ and related questions
- PhD (University of Virginia, ongoing, with Andrew Obus)
 - Algebraic and arithmetic geometry
 - Canonical rings of stacky curves
 - The unit equation in number fields and function fields

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I recently passed my second year proficiency exam, so I am just getting started on a couple projects!

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- (Baker) When K is a number field, there is an effective upper bound for number of solutions.

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- (Mason, Silverman) When $K = k(C)$ is the function field of a smooth projective curve, the S -unit equation has finitely many solutions (up to $u, v \in k^*$) and there is an effective upper bound.

Further Details: Unit Equations

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- (Baker) When K is a number field, there is an effective upper bound for number of solutions.
- (Mason, Silverman) When $K = k(C)$ is the function field of a smooth projective curve, the S -unit equation has finitely many solutions (up to $u, v \in k^*$) and there is an effective upper bound.
- Ambitious goal: reprove these results using “smallness” of the étale fundamental group $\pi_1^{\text{ét}}(X)$ where $X = C$ or $X = \text{Spec } \mathbb{Z}$.

Saito-Kurokawa Liftings and Congruence Primes

Huixi Li

Clemson University

UNCG Summer School in Computational Number Theory

May 29, 2017

Mixed-Level Saito-Kurokawa Liftings

Classical Saito-Kurokawa liftings:

$$S_{2k-2}(\mathrm{SL}_2(\mathbb{Z})) \xrightarrow{\cong} J_{k,1}^c(\mathrm{SL}_2^J(\mathbb{Z})) \xrightarrow{\cong} S_k^{\mathrm{Maa}\beta}(\mathrm{Sp}_4(\mathbb{Z})).$$

Ramakrishnan generalized this further:

$$S_{2k-2}(\Gamma_0(Nt)) \longrightarrow J_{k,t}(\Gamma_0^J(N)).$$

Zantout and Schmidt proved:

$$\begin{array}{ccccc} S_{2k-2}^{t,\mathrm{new}}(\Gamma_0(Nt)) & \xrightarrow{\text{Rama.}} & J_{k,t}^{c,\mathrm{new}}(\Gamma_0^J(N)) & \xrightarrow[\mathrm{DZ}]{\cong} & S_k^{\mathrm{DZ},\mathrm{new}}(\Gamma_N[t]) . \\ & & & \searrow & \\ & & & \cong & \\ & & & \text{Rep. Th.} & \end{array}$$

Theorem (Brown-L.)

The map given by Ramakrishnan is a Hecke equivariant isomorphism from $S_{2k-2}^{t,\mathrm{new}}(\Gamma_0(Nt))$ to $J_{k,t}^{cusp,\mathrm{new}}(\Gamma_0^J(N))$, where $N, t \in \mathbb{N}$ are odd square free integers such that $\mathrm{gcd}(N, t) = 1$.

Theorem (Hida 1981)

Let $f \in S_k(\Gamma_0(N))$ be a newform. Then p is a congruence prime for f if and only if $p \mid L^{\text{alg}}(k, \text{Sym}^2 f)$.

Theorem (Brown-Klosin 2016)

Let $f \in \mathcal{S}_{n,k}(\mathfrak{M})$ be an automorphic form on the unitary group $U(n, n)(\mathbb{A}_F)$, where F is a totally real field, K/F is a quadratic imaginary extension and $\mathfrak{M} \mid \mathfrak{N}$ are ideals of \mathcal{O}_F . Then p is a congruence prime of f if

$$\text{val}_p \left(\frac{\pi^{dn^2}}{\text{vol}(\mathcal{F}_{\mathcal{K}_{0,n}(\mathfrak{M})})} \overline{L^{\text{alg}}(2n + t/2, f, \xi; st)} \right) < 0.$$

Galois Groups of Eisenstein Polynomials over Local Fields

Jonathan Milstead

May 22, 2017

Ramification Polygons

① Definition: Newton polygon of $\frac{\varphi(\alpha x + \alpha)}{\alpha^n}$.

② One Segment (Greve): $\text{Gal}(\varphi) = G_1 \rtimes H$

$$\{t_{A,v} : (\mathbb{F}_p)^m \rightarrow (\mathbb{F}_p)^m : x \mapsto Ax + v \mid A \in H' \leq \text{GL}(m, p), v \in (\mathbb{F}_p)^m\}$$

③ Max Tame Subextension (Greve)

$$T = I \left({}^{e_1 e_0} \sqrt{(-1)^{v_1} \gamma_1^{b_1 n} \varphi_0}, \dots, {}^{e_\ell e_0} \sqrt{(-1)^{v_\ell} \gamma_\ell^{b_\ell n} \varphi_0} \right)$$

Blocks

1 Greve

$$\Delta_i = \{\alpha' \in \bar{K} \mid \varphi(\alpha') = 0 \text{ and } \nu_L(\alpha' - \alpha_1) \geq m_i + 1\}$$

2 Starting Group (Ex. 3 segments)

$$\text{Gal}(\varphi) \leq \text{Gal}(L_1/L_2) \wr (\text{Gal}(L_2/L_3) \wr \text{Gal}(L_3/\mathbb{Q}_p))$$

3 Residual Polynomial Classes (Milstead, Pauli)

$$\left\{ \begin{array}{l} \varphi(\alpha') = 0 \text{ and either} \\ \alpha' : \nu_L(\alpha' - \alpha_1) > m_i + 1 \text{ or} \\ \nu_L(\alpha' - \alpha_1) = m_i + 1 \text{ and } \frac{-1 + \frac{\alpha'}{\alpha_1}}{\alpha_1^{m_i}} \in \underline{\delta}\mathbb{F}_p \end{array} \right\}$$

Enumerating extensions of p -adic fields with given invariants

Sebastian Pauli

(joint work with Brian Sinclair)

University of North Carolina Greensboro

Generating Polynomials of Extensions of \mathbb{Q}_3 of degree 9

1. Totally ramified, thus Eisenstein 5085 Extensions
2. Valuation of Discriminant: 15 162 Extensions

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
3^3	0	0	0	0	0	0	0	0	0	0
3^2	*	*	*	*	*	*	*	*	*	0
3^1	$\neq 0$	0	0	*	0	0	*	$\neq 0$	*	0
3^0	0	0	0	0	0	0	0	0	0	1

$$3^{12} \cdot 2^2 = 2\,125\,764 \text{ Polynomials}$$

Generating Polynomials of Extensions of \mathbb{Q}_3 of degree 9

1. Totally ramified, thus Eisenstein 5085 Extensions
2. Valuation of Discriminant: 15 162 Extensions
3. Ramification polygon: $\{(1, 7), (3, 3), (9, 0)\}$ 108 Extensions

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
3^3	0	0	0	0	0	0	0	0	0	0
3^2	*	*	*	*	*	*	*	*	*	0
3^1	$\neq 0$	0	0	$\neq 0$	0	0	*	$\neq 0$	*	0
3^0	0	0	0	0	0	0	0	0	0	1

$$3^{11} \cdot 2^3 = 1\,417\,176 \text{ Polynomials}$$

Generating Polynomials of Extensions of \mathbb{Q}_3 of degree 9

1. Totally ramified, thus Eisenstein 5085 Extensions
2. Valuation of Discriminant: 15 162 Extensions
3. Ramification polygon: $\{(1, 7), (3, 3), (9, 0)\}$ 108 Extensions
4. Residual polynomials: $(2 + z^2, 1 + z^3)$ 27 Extensions

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
3^3	0	0	0	0	0	0	0	0	0	0
3^2	*	0	0	0	0	0	0	0	0	0
3^1	1	0	0	1	0	0	0	2	*	0
3^0	0	0	0	0	0	0	0	0	0	1

$3^2 = 9$ Polynomials

Generating Polynomials of Extensions of \mathbb{Q}_3 of degree 9

1. Totally ramified
2. Valuation of Discriminant: 15
3. Ramification polygon: $\{(1, 7), (3, 3), (9, 0)\}$, slopes $-2, -1/3$
4. Residual polynomials: $(2 + z^2, 1 + z^3)$

Each of the 27 extensions of \mathbb{Q}_3 with these invariants is generated by exactly one of the polynomials:

$$\begin{array}{lll} x^9 + 6x^7 + 3x^3 + 3 & x^9 + 3x^8 + 6x^7 + 3x^3 + 3 & x^9 + 6x^8 + 6x^7 + 3x^3 + 3 \\ x^9 + 6x^7 + 3x^3 + 12 & x^9 + 3x^8 + 6x^7 + 3x^3 + 12 & x^9 + 6x^8 + 6x^7 + 3x^3 + 12 \\ x^9 + 6x^7 + 3x^3 + 21 & x^9 + 3x^8 + 6x^7 + 3x^3 + 21 & x^9 + 6x^8 + 6x^7 + 3x^3 + 21 \end{array}$$

Each polynomial generates 3 distinct extensions.

- Member: *American Institute of Aeronautics and Astronautics*
- Member: *American Mathematical Society*
- Collaborating with Dr. Vladimir Golubev (ERAU, AFOSR) on *Multi-disciplinary Design Optimization of Synthetic Jet Actuators for Transitional Boundary Layer Separation Control*
- First gained interest in **Modular Forms** exactly one year ago.
- **Research Interest:** Investigate the occurrence of doubly-periodic vortex soliton solutions of coupled nonlinear hyperbolic partial differential equations.
- **Research Interest:** Arithmetic on groves of planar binary trees using the Loday-type dendriform dialgebra.
- **Research Interest:** Symbolic computation and term rewriting using abstract syntax trees for expressions.

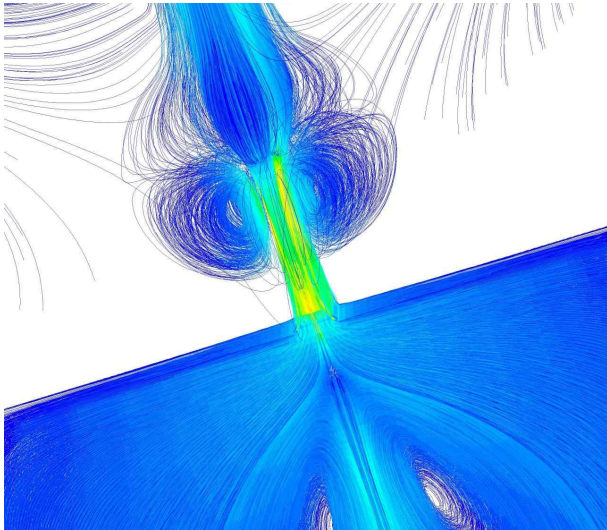


Figure: Vena contracta visible near orifice of asymmetric flow synthetic jet actuator, 3D Reynolds Averaged Navier-Stokes PDE

An introduction to my research interest

Manami Roy

University of Oklahoma

May 22, 2017

UNCG Summer School in Computational Number Theory 2017

My research interest

- Automorphic forms and automorphic representation
- Classical modular forms and Siegel modular forms
- Automorphic L-functions
- Elliptic curves
- The Langlands program

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- Master's project: The Ramanujan's conjectures and L-functions corresponding to cusp forms.

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Current Research:

- I am investigating level of the Siegel modular forms under different congruence subgroups constructed via the sym^3 lifting.

The sym^3 lifting and level of Siegel modular forms

$$\begin{array}{ccc} \pi & \longleftrightarrow & f \in S_k(\Gamma_0(N)) \\ \downarrow \text{functoriality(!)} & & \downarrow \\ II & \longleftrightarrow & F \in S_{k+1, k-2}(??) \end{array}$$

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f : a cuspidal eigen-newform of weight k and level N .

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II : a cuspidal automorphic representation of $\text{GSp}(4, \mathbb{A})$.

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Goal: To find the level of F under appropriate congruence subgroups.

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- $K(p^n) = \left\{ k \in \text{GSp}(4, \mathbb{Q}_p) : k \in \begin{bmatrix} \mathbb{Z}_p & \mathbb{Z}_p & \mathbb{Z}_p & p^{-n} \\ p^n & \mathbb{Z}_p & \mathbb{Z}_p & \mathbb{Z}_p \\ p^n & \mathbb{Z}_p & \mathbb{Z}_p & \mathbb{Z}_p \\ p^n & p^n & p^n & \mathbb{Z}_p \end{bmatrix} \text{ and } \det(k) \in \mathbb{Z}_p^\times \right\}$

Number Theory Summer School

James Rudzinski

University of North Carolina at Greensboro

May 22, 2017

Symmetric Chain Decomposition

Definition (Symmetric chain decomposition)

A symmetric chain decomposition of B_n is a partition of B_n into symmetric chains.

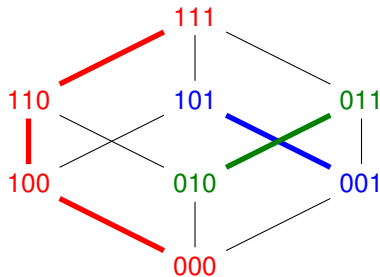


Figure: Symmetric chain decomposition of B_3

Symmetric Chain Decomposition

We were able to organize the initial strings in a tree so that each string could also be attained in an efficient way by only adding ones. We can recursively generate the initial string of each chain along with the indices of the zeros that are to be changed to ones.

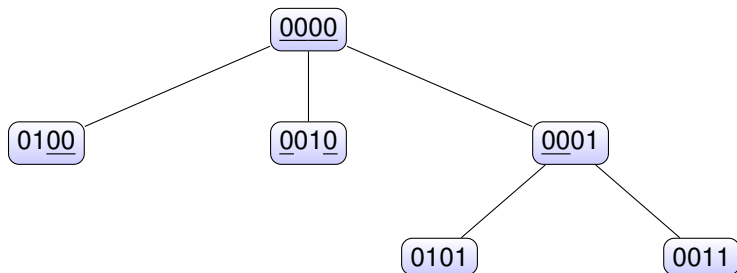


Figure: Tree of initial strings for the symmetric chain decomposition of B_4 .

Introduction

Sandi Rudzinski

Department of Mathematics and Statistics
University of North Carolina at Greensboro

UNCG Number Theory Summer School 2017

Resolvent Polynomials

- The roots of a resolvent polynomial are created by plugging in the roots of one polynomial into variations of a multivariate polynomial.
- My thesis focused on computing special types of resolvent polynomials without the use of root approximations.

Filip SAIDAK

Department of Mathematics

UNC Greensboro

RESEARCH INTERESTS

Analytic, Probabilistic and Elementary Number Theory

1. Prime numbers

- general distribution
- special forms

2. Riemann ζ -function

- properties of zeros
- non-vanishing
- higher derivatives
- monotonicity
- Dirichlet L-functions

3. Arithmetic functions

- probabilistic results
- special values

Introduction & Research Interests

Christian Steinhart

Universität des Saarlandes

22nd May 2017, Greensboro

Teichmüller-Theory

- What are the possible complex structures on a topological surface S ?

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- Given a (quadratic) differential on such a structure yields a flat surface, i.e. polygons with parallel sides of equal length identified.

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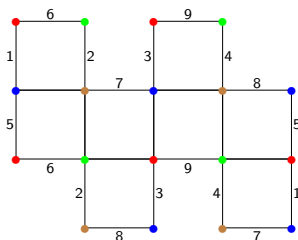


Figure: eierlegende Wollmilchsau

Teichmüller-Theory

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Projects:

- Embedding of Teichmüller disks of Origamis, i.e. coverings of the once-punctured torus, into the Outer Space CV_n .

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Projects:

- Embedding of Teichmüller disks of Origamis, i.e. coverings of the once-punctured torus, into the Outer Space CV_n .
- Finding similar embeddings of more general Teichmüller disks.

Introduction

Makoto Suwama

University of Georgia

UNCG Summer School, 2017

Just finished my first year in graduate school at UGA.

Interests

- (Algebraic) Number Theory
- Arithmetic Geometry
- Algebraic Geometry
- Representation Theory

Plan to find an advisor and a thesis topic this summer.

Undergraduate thesis

Suppose

- K is a number field
- E/K is an elliptic curve defined over K

Then

- $E(K) \cong \mathbb{Z}^r \times E(K)_{Tors}$

Want to compute $E(K)/nE(K)$, for some $n \in \mathbb{Z}$. (HARD)

\implies Instead, compute $\text{Sel}^{(n)}(E/K)$.

$$0 \longrightarrow E(K)/nE(K) \longrightarrow \text{Sel}^{(n)}(E/K) \longrightarrow \text{III}(E/K) \longrightarrow 0$$

For my undergraduate thesis, I studied 2-descent algorithm which computes $\text{Sel}^{(2)}(E/K)$.

Stark's Conjecture as it relates to Hilbert's 12th Problem

Brett A. Tangedal

University of North Carolina at Greensboro, Greensboro NC, 27412, USA
batanged@uncg.edu

May 22, 2017



Let F be a real quadratic field, \mathcal{O}_F the ring of integers in F , and \mathfrak{m} an integral ideal in \mathcal{O}_F with $\mathfrak{m} \neq (1)$. There are two infinite primes associated to the two distinct embeddings of F into \mathbb{R} , denoted by $\mathfrak{p}_\infty^{(1)}$ and $\mathfrak{p}_\infty^{(2)}$. Let $\mathcal{H}_2 := H(\mathfrak{mp}_\infty^{(2)})$ denote the ray class group modulo $\mathfrak{mp}_\infty^{(2)}$, which is a finite abelian group.

Given a class $\mathcal{C} \in \mathcal{H}_2$, there is an associated partial zeta function $\zeta(s, \mathcal{C}) = \sum \mathfrak{N}\mathfrak{a}^{-s}$, where the sum runs over all integral ideals (necessarily rel. prime to \mathfrak{m}) lying within the class \mathcal{C} . The function $\zeta(s, \mathcal{C})$ has a meromorphic continuation to \mathbb{C} with exactly one (simple) pole at $s = 1$. We have $\zeta(0, \mathcal{C}) = 0$ for all $\mathcal{C} \in \mathcal{H}_2$, but $\zeta'(0, \mathcal{C}) \neq 0$ (if certain conditions are met).

First crude statement of Stark's conjecture: $e^{-2\zeta'(0, \mathcal{C})}$ is an algebraic integer, indeed this real number is conjectured to be a root of a palindromic monic polynomial

$$f(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_2x^2 + a_1x + 1 \in \mathbb{Z}[x].$$

For this reason, $e^{-2\zeta'(0, \mathcal{C})}$ is called a “Stark unit”. By class field theory, there exists a ray class field $F_2 := F(\text{mp}_\infty^{(2)})$ with the following special property: F_2 is an abelian extension of F with $\text{Gal}(F_2/F) \cong \mathcal{H}_2$. Stark's conjecture states more precisely that $e^{-2\zeta'(0, \mathcal{C})} \in F_2$ for all $\mathcal{C} \in \mathcal{H}_2$.

This fits the general theme of Hilbert's 12th problem: Construct analytic functions which when evaluated at “special” points produce algebraic numbers which generate abelian extensions over a given base field.

Introduction

Debbie Demet White

Department of Mathematics and Statistics
University of North Carolina at Greensboro
to be a mathematician...

May 2017

About me

- ▶ I graduated from Middle East Technical University from Turkey in 1992 then I worked as a high school teacher in many public and private schools for almost 20 years.
- ▶ I moved to the US for my son some health problems, the plan was to go back after his things is done, but when I realized I am still here and will continue to live, I decided to go back to my teaching career.

About me

- ▶ Now I want to enjoy beauty of mathematics. I think of mathematics as a language; studying the grammar can be tedious, boring and frustrating, but it is necessary to use it in whichever way we may decide. Understanding and appreciating the different genres of literature requires a good knowledge of the basic rules of the language in which they are expressed. When we enjoy a poem or a short story we never think about the grammar behind it, if we did it would spoil our enjoyment; instead we marvel at the finished product. Mathematics can be viewed in a similar way: the long hours spent working calculus problems are the way to make the basic rules become so natural and instinctive that we do not need to think about them. We can concentrate on other, more exciting and challenging problems.
- ▶ I am at the beginning of this journey.
- ▶ I excited being here to learn more. . .

Introductory Talk

Luciena Xiao

Department of Mathematics
California Institute of Technology

UNCG Summer School in Computational Number Theory 2017

Hypersymmetric Abelian Varieties

Definition (Hypersymmetric Abelian Variety)(Chai and Oort, 2006; Zong, 2008)

Let (Γ) be a finite dimensional semi-simple algebra over \mathbb{Q} with positive involution. Let A be an abelian variety over an algebraically closed field k of characteristic p . A is Γ -hypersymmetric if the natural map

$$\text{End}_{\Gamma}(A) \otimes_{\mathbb{Q}} \mathbb{Q}_p \rightarrow \text{End}_{\Gamma \otimes_{\mathbb{Q}} \mathbb{Q}_p}(A[p^{\infty}])$$

is a bijection.

Theorem

(Zong, 2008) *An effective Γ -linear polarized isocrystal M is isomorphic to the Dieudonné isocrystal of a Γ -hypersymmetric abelian variety A if and only if M underlies an (S) -restricted partitioned isocrystal.*

A Special Case: $\Gamma = \mathbb{Q}$

Definition

An abelian variety A over an algebraically closed field is hypersymmetric if the natural homomorphism $End(A) \otimes_{\mathbb{Z}} \mathbb{Z}_p \rightarrow End(A[p^\infty])$ is an isomorphism.

Theorem

(Chai and Oort, 2006) For every symmetric Newton polygon ζ and every prime number p there exists a hypersymmetric abelian variety A over $\overline{\mathbb{F}}_p$ with Newton polygon equal to ζ .

Application

Chai used hypersymmetric points to show a conjecture by F. Oort that every prime-to- p Hecke orbit in the moduli space \mathcal{A}_g of principally polarized abelian varieties over $\overline{\mathbb{F}}_p$ is dense in the leaf containing it.

Applications of Voronoi to Automorphic Forms

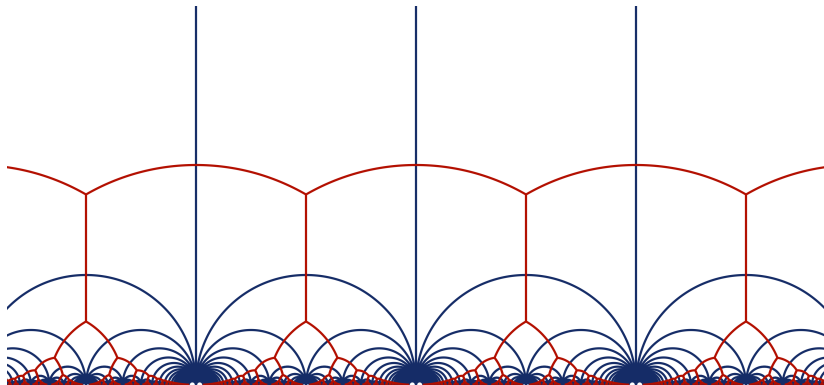
Dan Yasaki

The University of North Carolina Greensboro

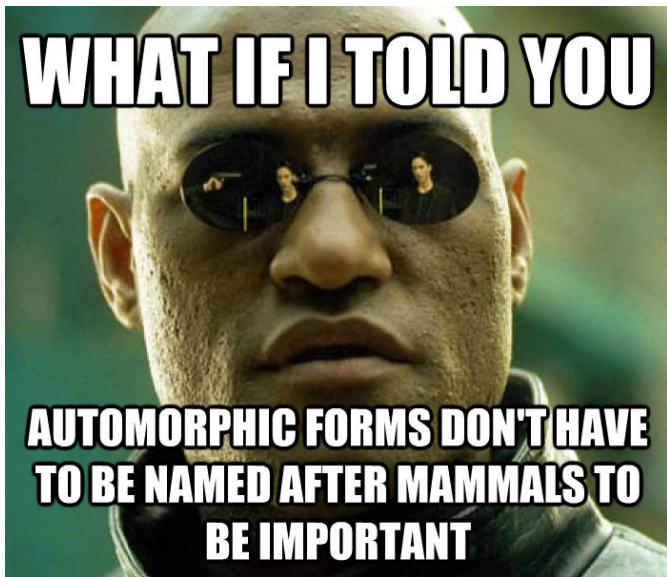
UNCG Summer School in Computational Number Theory
Modular Forms: May 22–26, 2017

Perfect forms and tessellations: $\mathbf{G} = \mathrm{SL}_2 / \mathbb{Q}$

$$\phi(x, y) = x^2 - xy + y^2, \quad M(\phi) = \left\{ \pm \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \pm \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \pm \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$



Automorphic forms



Yuan Yan

Department of Mathematics and Statistics
University of Massachusetts Amherst
May 21, 2017

Introduction

- I just finished my first year graduate study at University of Massachusetts Amherst
- I got master's degree in mathematics at The University of Texas at Austin

- Generally speaking, I am interested in number theory. In particular, the theory of algebraic number, elliptic curves and modular forms