Computational Aspects of Buildings
Further projects

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1 Project 1

The second paper of CMSZ contains all the triangular presentations up to equivalence of the finite projective plane over $\mathbf{F}_2$ and $\mathbf{F}_3$, denoted $\mathrm{PG}(2,2)$ and $\mathrm{PG}(2,3)$. Since the Hughes plane contains $\mathrm{PG}(2,3)$ as a subplane and since the triangular presentation of Radu is an extension of a triangular presentation of $\mathrm{PG}(2,3)$, it would be interesting, and probably fast, to find all extended triangular presentations. In the Appendix of CMSZ II, the presentation appearing in Radu's presentation is number 14.1. However this lattice is not coming from the algebraic group $\mathrm{PGL}_3(F)$ over the local fields $F = \mathbf{Q}_3$ or $\mathbf{F}_3((X))$. This is why I suggest starting with the presentation coming from these groups, see Theorem 2 in CMSZ II for the list.

~~I would particularly insist on $\mathbf{F}_3((X))$ for the following reason. The Hughes plane is actually associated to a finite near-field k of order 9 over $\mathbf{F}_3$, so possibly $k((X))$ could be a meaningful near-field for which a $\widetilde{A}_2$-building could exists as the Bruhat-Tits building associated to the hypothetical object named $\mathrm{PGL}_3(k((X)))$.~~ This applies only to the Hall plane and I misremembered the construction of the Hughes plane.

Contact: Nicolas Radu

Reference:

- Cartwright, Mantero, Steger, Zappa: Groups acting simply transitively on the vertices of a building of type $\tilde{A}_2$, II: The cases q=2 and q=3.
- Cartwright, Mantero, Steger, Zappa: Groups acting simply transitively on the vertices of a building of type $\tilde{A}_2$, I

2 Project 2

Another project suggested by Alina Vdovina would be to look at hyperbolic building of type $\pi/4, \pi/4, \pi/4$. Indeed the rank two residues are spherical buildings of type C_2, i.e. generalized 4-gones, also called generalized quadrangles. The regularity parameters are would have to be all equal to a thickness $q \geq 2$ if one hopes to obtain a transitive action on the vertices. Similar theory of triangular presentation applies.

By a result of Ballman-Brin, if q is greater than 7, the group associated to a triangular presentation has Property (T) and would be an interesting example of a hyperbolic Kazhdan group. Caprace recently exhibited such a group with 4 generators and 16 relations acting cocompactly on a hyperbolic building. However, the latter is not vertex homogeneous, which would be the case here.

Contact: Alina Vdovina

3 Project 3

The score algorithm of Radu, Algorithm 1 in the paper, has worked in practice for all the cases tested by Radu or ourselves. It is however not clear that there is a theoretical reason for that. Recall that it consists in searching for a covering by triangles of the oriented graph G_λ. Radu's algorithm looks at edges one by one and if an edge is contained in exactly one triangle, it removes that triangle. If an edge sits in two or no triangle, it leaves it alone and tests other edges. Two questions are therefore natural:

- Write an algorithm that finds covers by disjoint triangles of an oriented graph of the form G_λ.
- Show that the algorithm of Radu theoretically works.