# UNCG Clock Talk

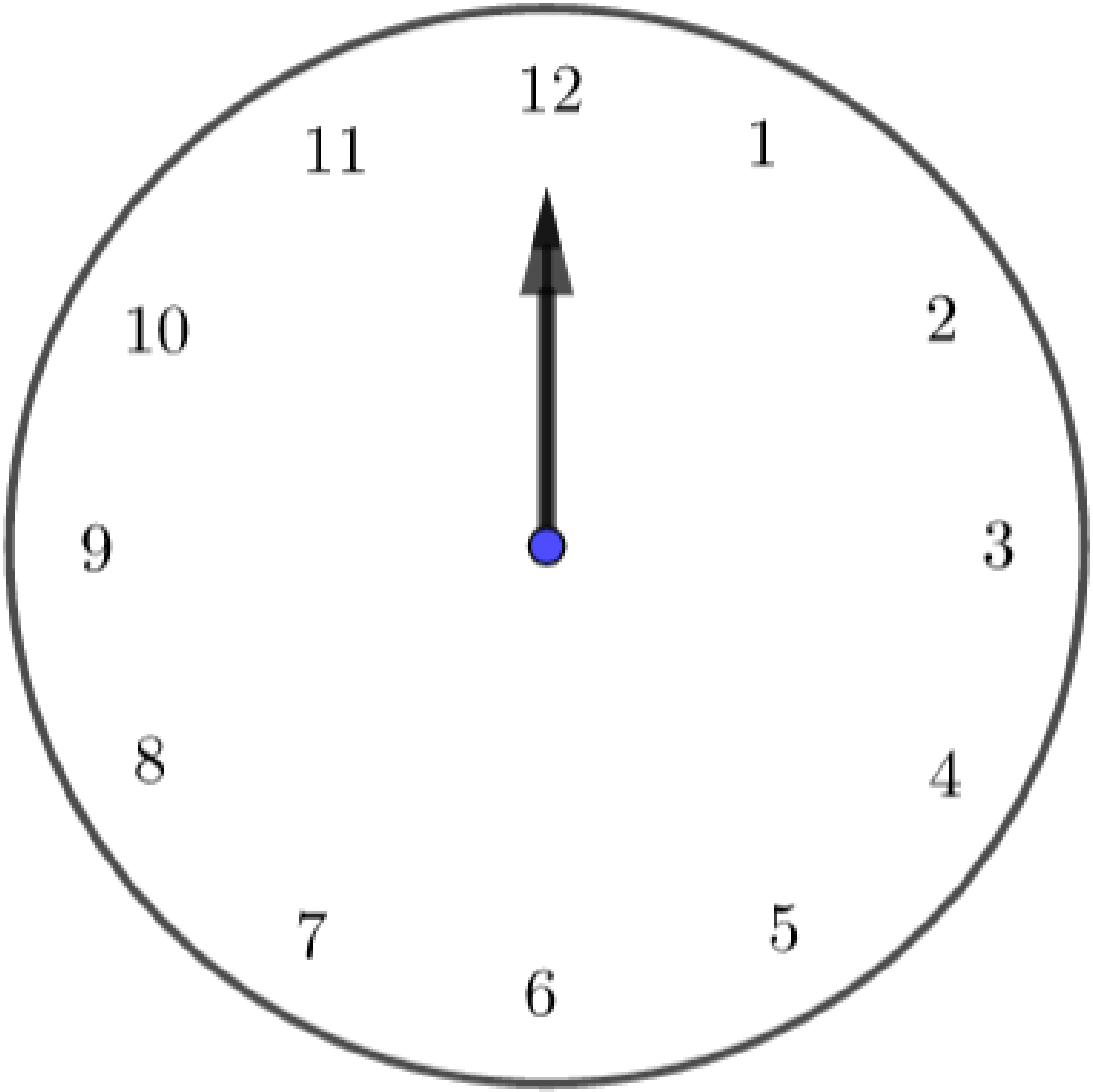
Chris Ratigan

Tufts University

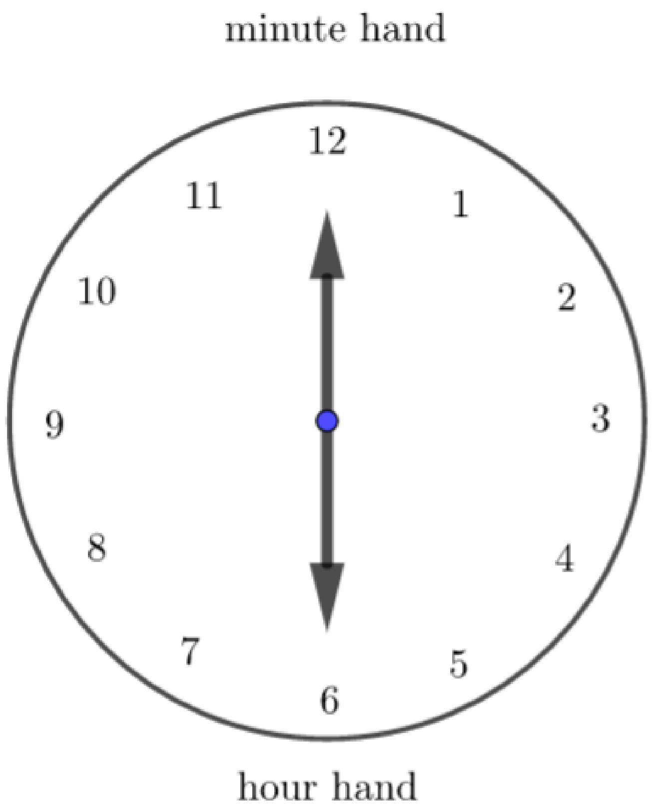
# Clock Problem

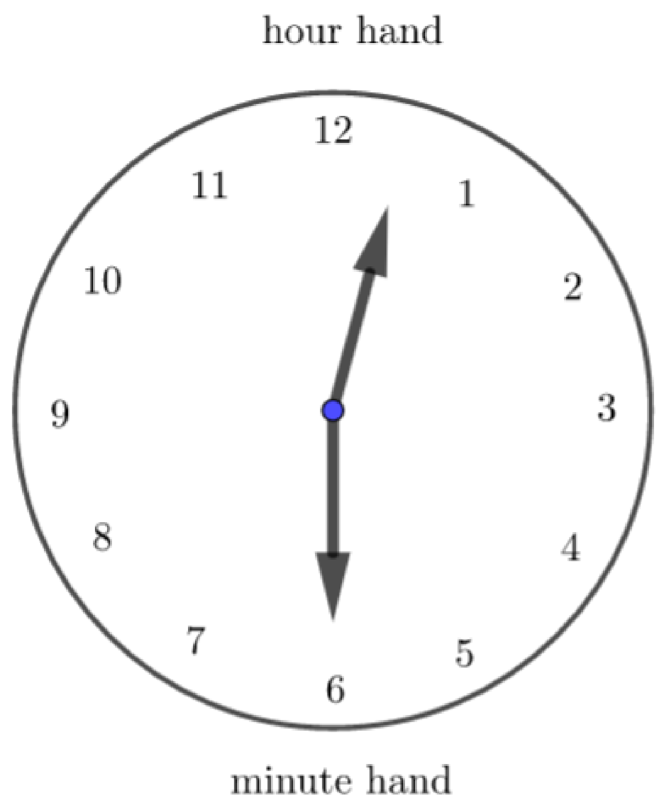
A clock has minute hand and hour hand which are distinct, but indistinguishable. Most of the time, you can tell what time it is.
**Question:** How many times can you not tell what time it is?

# 12:00



# 6:00 vs. 12:30

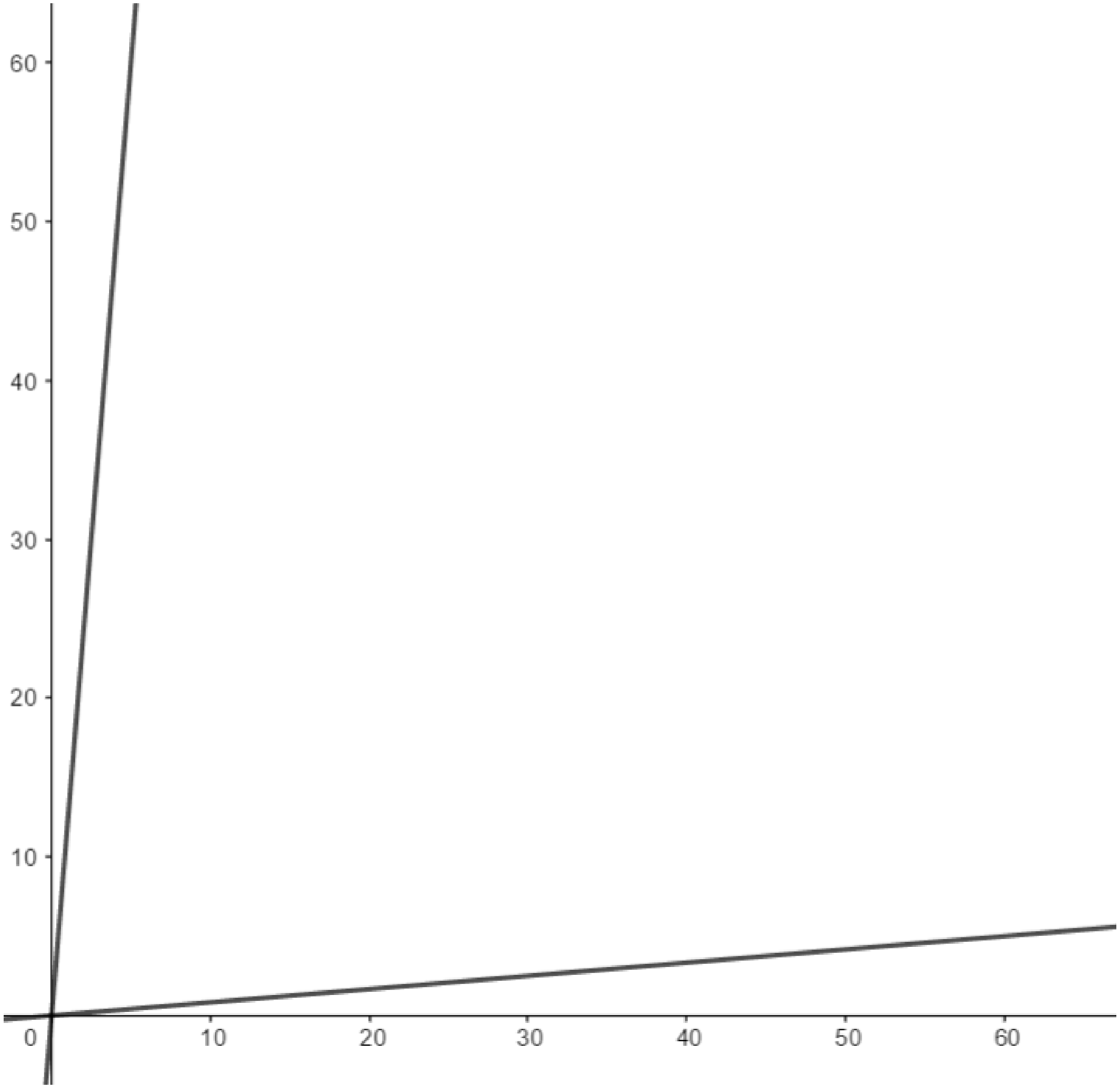

minute hand
12
1
2
3
4
5
6
7
8
9
10
11
hour hand


hour hand
12
1
2
3
4
5
6
7
8
9
10
11
minute hand

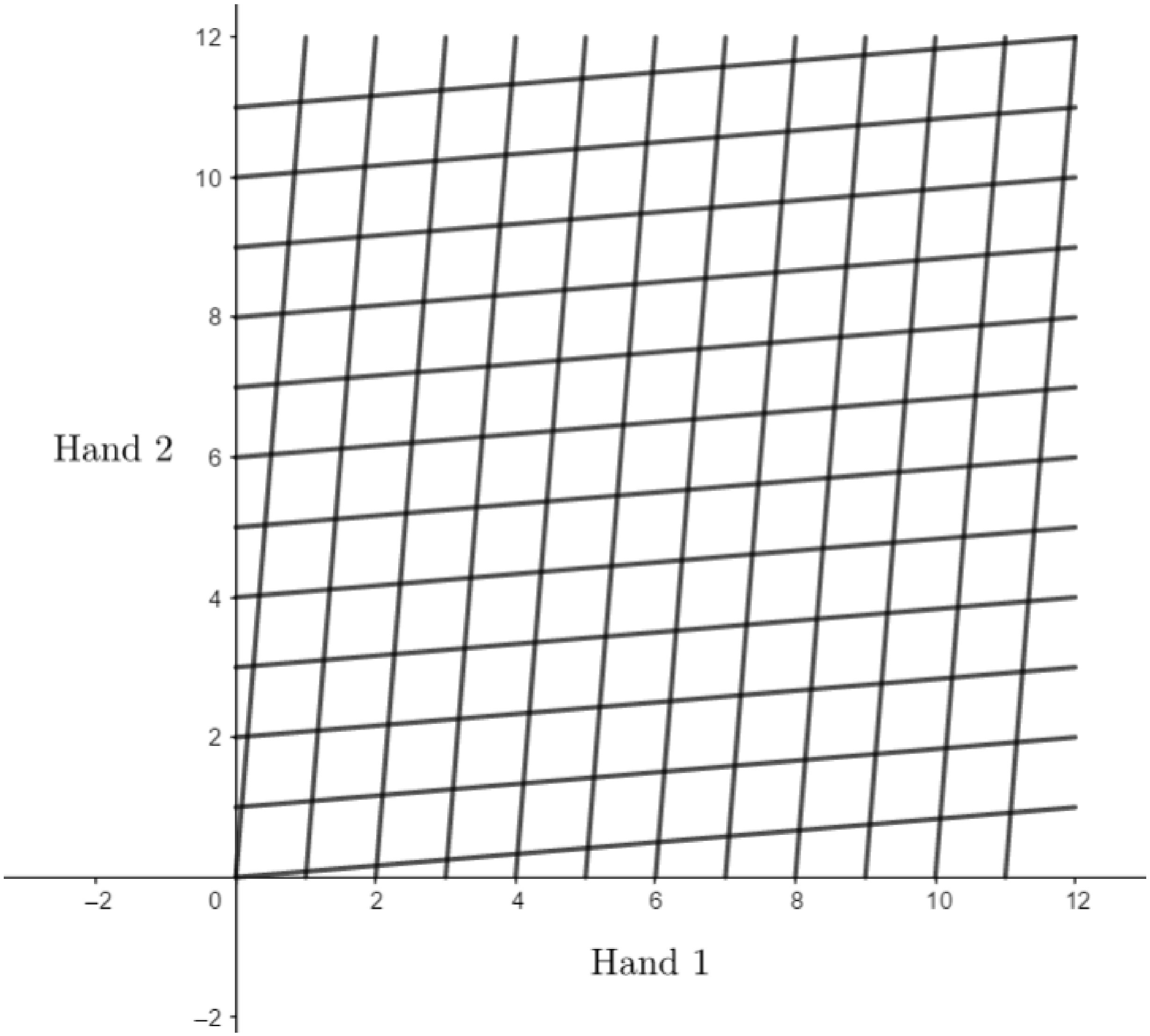
# Ambiguous?

**Fact 1** A time is ambiguous if interchanging hands creates a new time.
**Fact 2** The Minute hand goes around 12 times the speed of the hour hand.

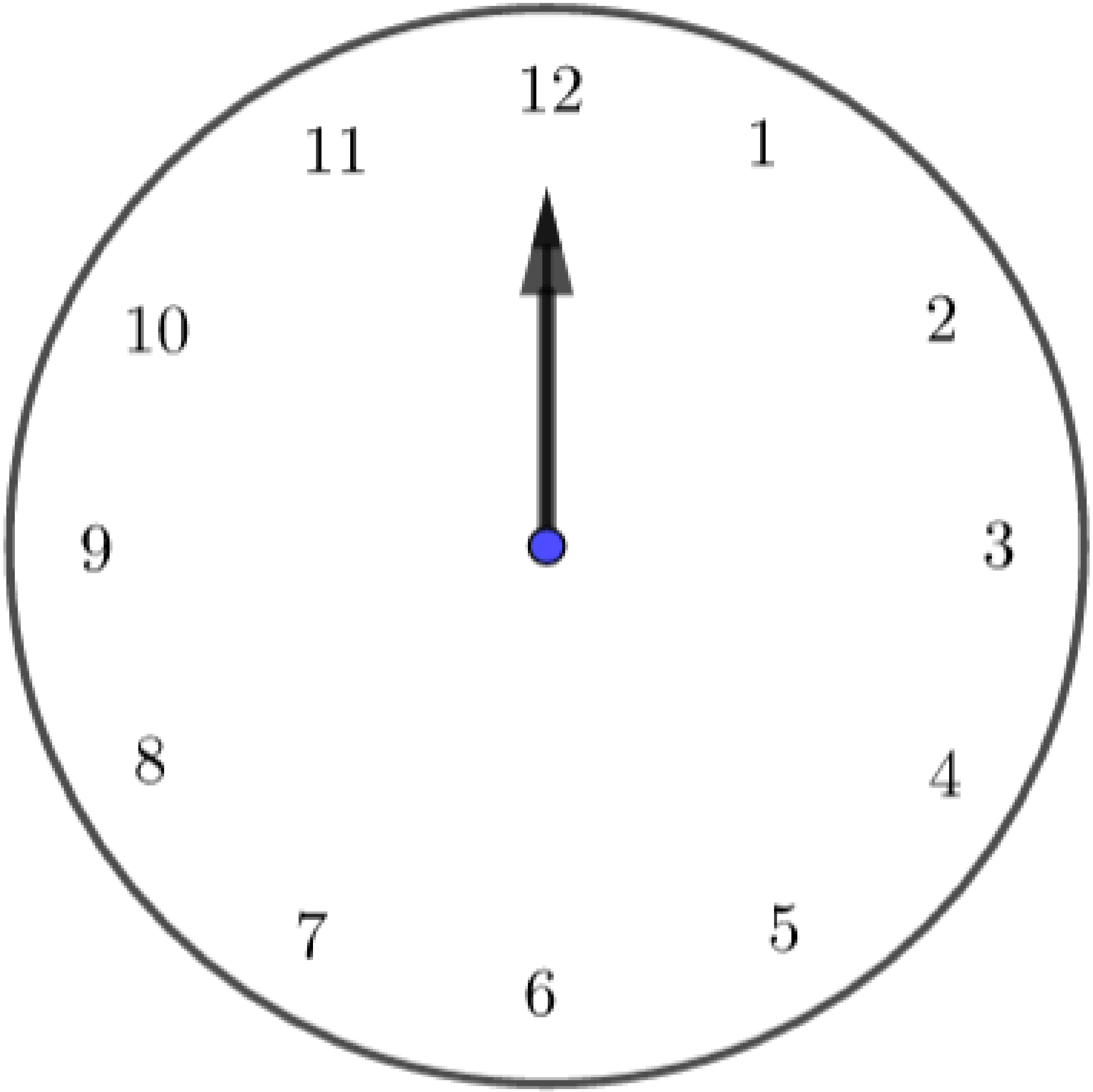
# Related Rates?



# Solution



## 12:00



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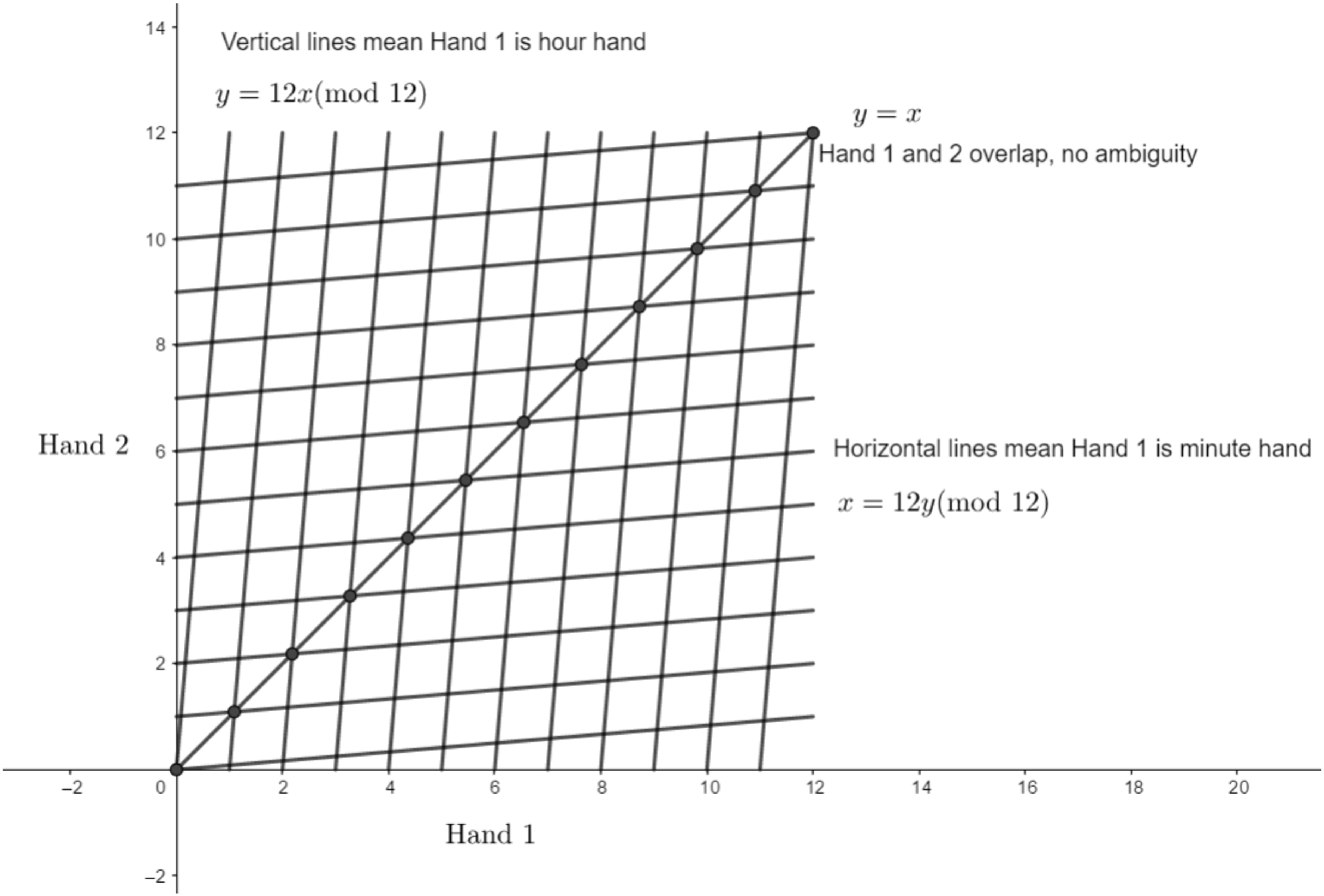
Vertical lines mean Hand 1 is hour hand
$y = 12x (\text{mod } 12)$
$y = x$
Hand 1 and 2 overlap, no ambiguity
Horizontal lines mean Hand 1 is minute hand
$x = 12y (\text{mod } 12)$
Hand 2
Hand 1
14
12
10
8
6
4
2
0
−2
2
4
6
8
10
12
14
16
18
20

# Epilogue

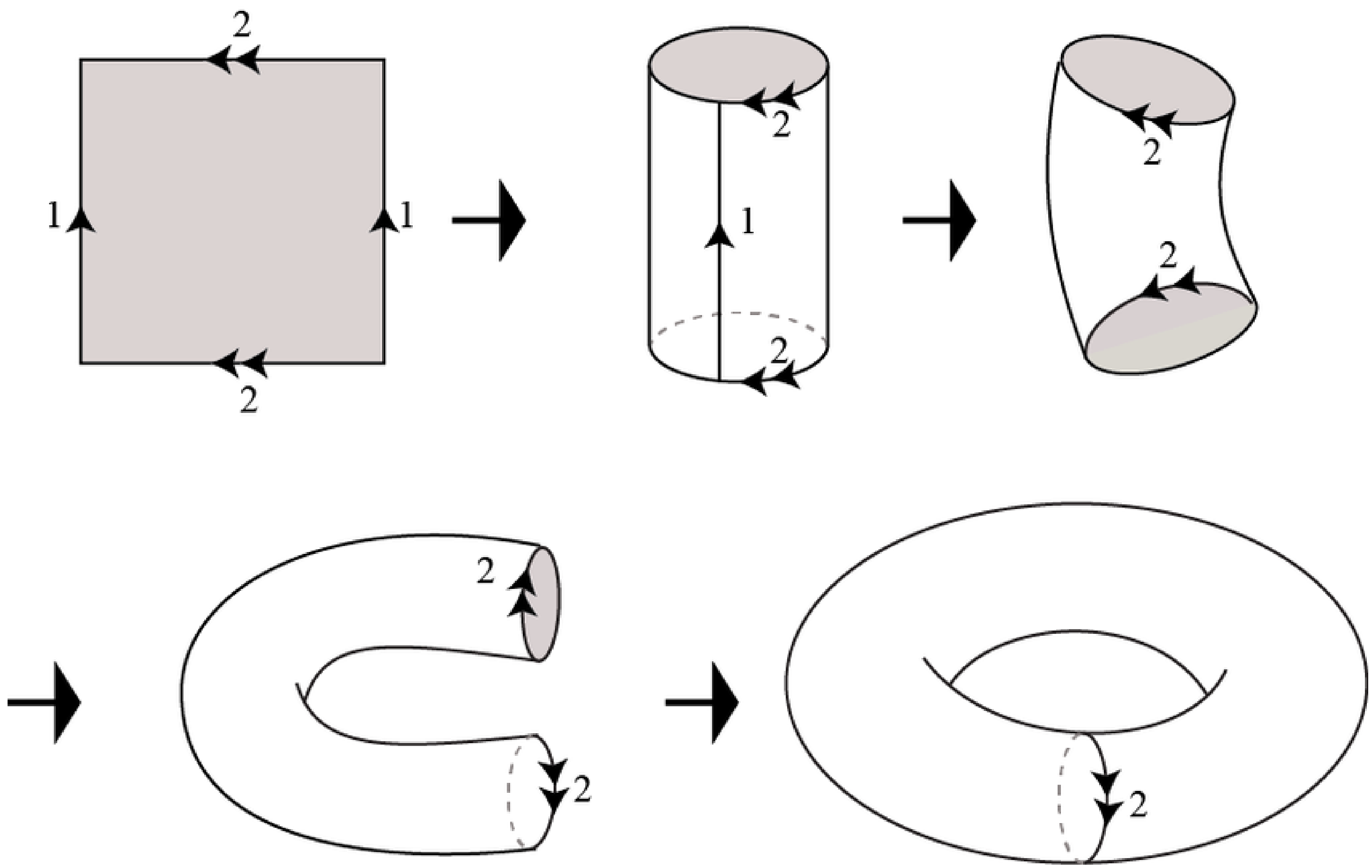


Image from "Survey of Graph Embeddings Into Compact Surfaces" on Researchgate

Thank You!

# Short presentations of $A_n$ and $S_n$

Peter Huxford

Supervisor: Professor Eamonn O'Brien

The University of Auckland,
New Zealand

June 28, 2019

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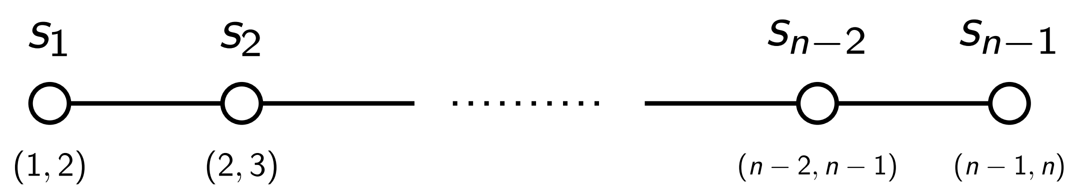
**Example**

$$D_n = \langle r, s \mid r^n = s^2 = (rs)^2 = 1 \rangle$$

2 generators. 2 relations. Word length: $O(n)$. Bit-length: $O(\log n)$.

The symmetric group is a Coxeter group.

$$S_n = \langle s_1, \ldots, s_{n-1} \mid s_i^2 = 1,\ (s_{i-1}s_i)^3 = 1,$$
$$(s_is_j)^2 = 1 \text{ if } |i-j| \geq 2 \rangle.$$

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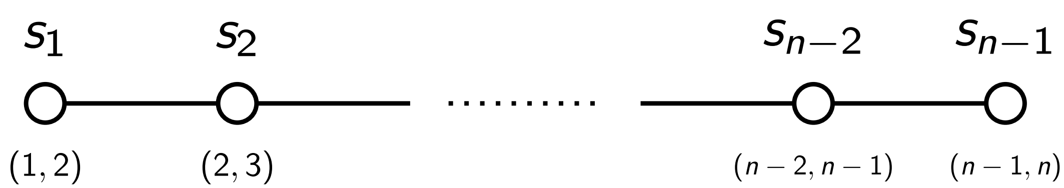


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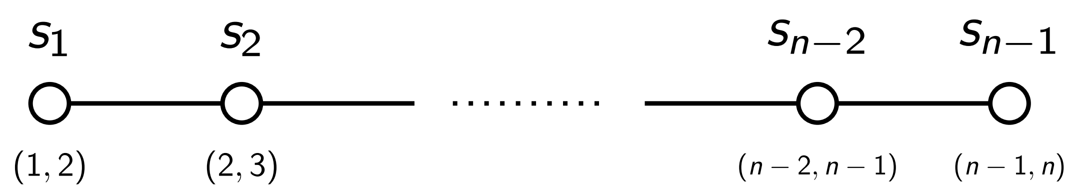


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There is a presentation of $A_{n+2}$ due to Carmichael (1923) with similar measurements, on the generators $(i, n+1, n+2)$ for $i = 1, \ldots, n$.

# Recent Improvements

The best bit-length possible has been achieved.

**Theorem (Bray, Conder, Leedham-Green, O'Brien, 2011)**

*$A_n$ and $S_n$ have presentations with a uniformly bounded number of generators and relations, and bit-length $O(\log n)$.*

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A stronger result has also been shown.

**Theorem (Guralnick, Kantor, Kassabov, Lubotzky, 2011)**

*$A_n$ and $S_n$ have 3-generator 7-relator presentations of bit-length $O(\log n)$.*

# What I have done

There are errors in (GKKL, 2011) regarding the presentations of $A_n$ and $S_n$.

These are now fixed.

See my honours dissertation (2019) and my GitHub for supporting code.
`https://github.com/pjhuxford/short-presentations`

# Lightning Talk–UNCG Computational Aspects of Buildings Summer School 2019

Richard Mandel

Stevens Institute of Technology

June 28, 2019

# About me

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- Also lived in UK, Romania and Israel as a child.

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# Current research (with Alexander Ushakov)

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- The Diophantine Problem (DP) for spherical equations over $BS(m, n)$ is the following decision problem. Given a *spherical equation* $W$:

$$w_1^{-1} c_1 w_1 \cdots w_k^{-1} c_k w_k = 1,$$

with unknowns $w_i$ and constants $c_i$, is it decidable whether or not $W$ has a solution?

# Current research

- Easy cases:
  We have $BS(1,1) \cong \mathbb{Z} \times \mathbb{Z}$ and $BS(1,-1) \cong \mathbb{Z} \rtimes \mathbb{Z}$; in these cases it can be shown that the problem is decidable in polynomial time.

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- NP-hardness can be proved by exhibiting a reduction of the *3-partition problem*.
- Remains to show that DP$\in$NP.

# Current research

- We hope to generalize methods to a wider class of groups (e.g. generalized Baumslag-Solitar groups).

# Thank you!

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