Low dimensional Euclidean buildings: II

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Table of Contents

Classification: Irreducibility

Let (W, S) be a Coxeter system. (Draw random Coxeter diagram with at least 2 connected components)

- Connected components: $I = J_1 \sqcup \cdots \cup J_n$ and $S = S_1 \sqcup \cdots \sqcup S_n$.
- The subgroups $W_{J_k} = \langle S_k \rangle$ pairwise commute.
- $W \cong W_1 \times \cdots \times W_n$ as groups.
- $(W, S) \cong (W_1, S_1) \times \cdots \times (W_n, S_n)$ as Coxeter systems.

Classification: Irreducibility

Let Δ be a building of type (W, S) and fix a chamber $c \in \mathcal{C}(\Delta)$.

- Connected components: $I = J_1 \sqcup \cdots \cup J_n$ and $S = S_1 \sqcup \cdots \sqcup S_n$.
- Let Δ_k denote the J_k-residue containing c, a building of type (W_k, S_k).
- The product chamber system $\Delta_1 \times \cdots \times \Delta_n$ is a chamber system over I where, for $i \in J_k$, the incidence is given by

  $$(c_1, \ldots, c_n) \sim_i (d_1, \ldots, d_n)$$

  if and only if $c_k \sim_i d_k$ and $c_\ell = d_\ell$ for all $\ell \neq k$.

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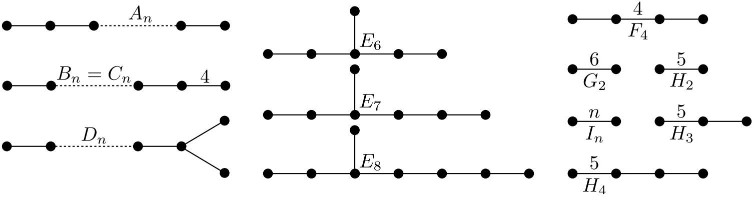
Theorem

The product $\Delta_1 \times \cdots \times \Delta_n$ is a building isomorphic to Δ (with $\sigma = \mathrm{id}$).

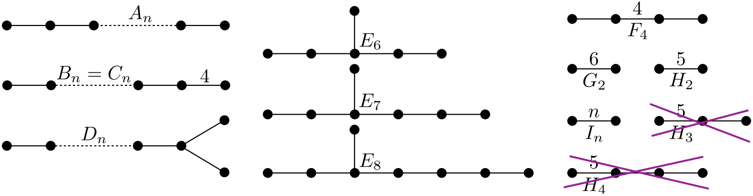
Classification: Spherical diagrams



Coxter (1934): all irreducible spherical Coxeter groups

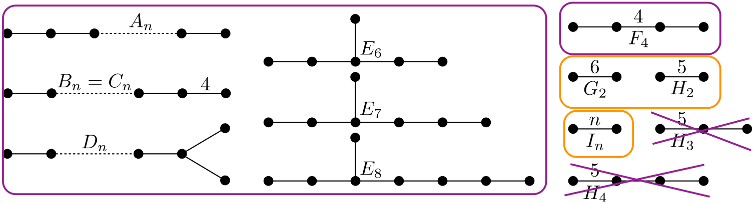
(wikipedia)

Classification: Spherical diagrams



Ronan-Tits: no building thick building of type H_3 or H_4.
(wikipedia)

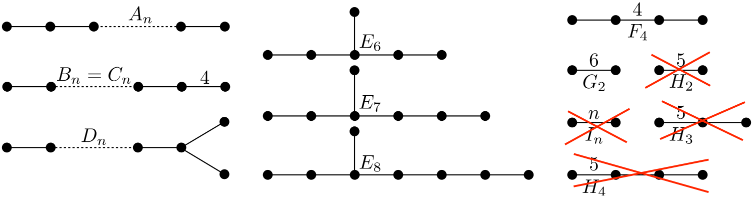
Classification: Spherical diagrams



Tits spherical classification: all buildings with diagram of rank ≥ 4.

(wikipedia)

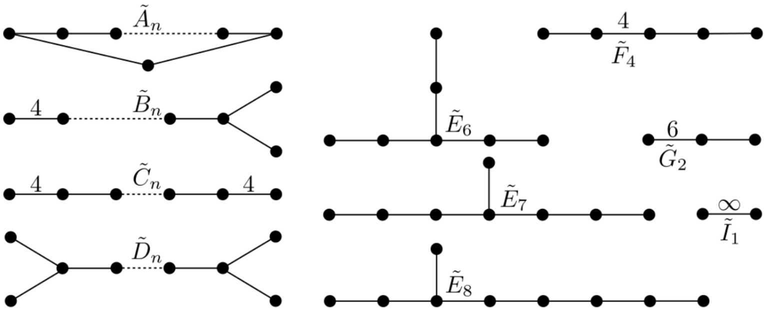
Classification: Spherical diagrams



No root system associated, hence no Euclidean extension.

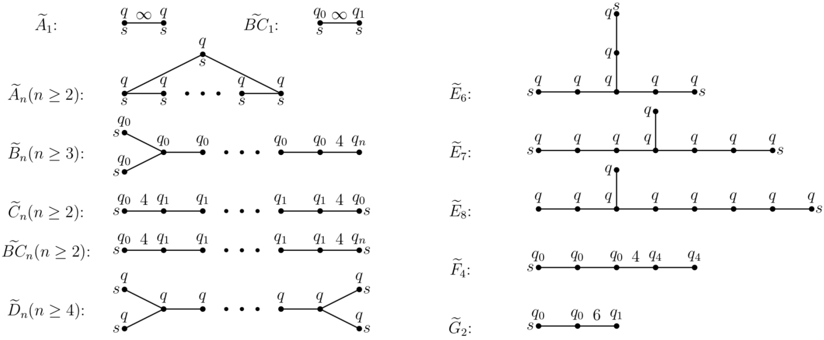
(wikipedia)

Classification: Euclidean diagrams



One edges added when Euclidean reflection groups exist.
(wikipedia)

Classification: Euclidean diagrams

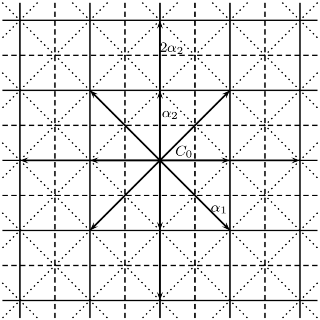


Classification with regularity parameters.

(Parkinson's thesis)

Classification: Euclidean diagrams

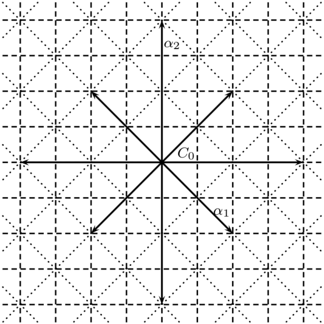
$(R = BC_2)$. Take $E = \mathbb{R}^2$, $\alpha_1 = e_1 - e_2$ and $\alpha_2 = e_2$. Then $B = \{\alpha_1, \alpha_2\}$ and $R^+ = \{\alpha_1, \alpha_2, \alpha_1 + \alpha_2, \alpha_1 + 2\alpha_2, 2\alpha_2, 2\alpha_1 + 2\alpha_2\}$.



(Parkinson's thesis)

Classification: Euclidean diagrams

($R = C_2$). Take $E = \mathbb{R}^2$, $\alpha_1 = e_1 - e_2$ and $\alpha_2 = 2e_2$. Then $B = \{\alpha_1, \alpha_2\}$ and $R^+ = \{\alpha_1, \alpha_2, \alpha_1 + \alpha_2, 2\alpha_1 + \alpha_2\}$ (see Example 3.1.2).



(Parkinson's thesis)

Classification

Tits' classification:

- Euclidean buildings of rank at least 4 are Bruhat-Tits buildings associated with an algebraic group $G(F)$ over a local field F and the automorphism group "is" $\mathrm{Aut}(\Delta) = G(F)$

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- $\mathrm{SL}_n(\mathbb{Q}_p)$