

Name: \_\_\_\_\_ Academic Integrity Signature: \_\_\_\_\_

*I have abided by the UNCG Academic Integrity Policy.***Read all of the following information before starting the exam:**

- It is to your advantage to answer ALL of the 20 questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- *Budget your time!*

Page:	1	2	3	4	5	Total
Points:	25	20	25	15	15	100
Score:						

1. (5 points) Compute the limit  $\lim_{x \rightarrow 1^-} \frac{x}{x^2 - 2x + 1}$ .
- A.  $\infty$    B.  $-\infty$    C. 1   D.  $\frac{1}{2}$    E. None of the above.

2. (5 points) Compute  $\frac{dy}{dx}$ , where  $y = 3x^2 - 7\sqrt{x} - \frac{4}{x^2}$ .

A.  $\frac{dy}{dx} = 6x - \frac{7}{2}x^{-1/2} + 8x^{-3}$

B.  $\frac{dy}{dx} = 6x - 7x^{1/2} + 8x^{-3}$

C.  $\frac{dy}{dx} = \frac{6x - \frac{7}{2}x^{-1/2}}{2x}$

D.  $\frac{dy}{dx} = -\frac{7}{2}x^{-1/2}$

E.  $\frac{dy}{dx} = 6x + 7x^{1/2} + 8x^{-1}$

F. None of the above.

3. (5 points) An object moves along the  $y$ -axis (marked in feet) so that its position at time  $t$  (in seconds) is given by

$$s(t) = 8t^3 - 3t^2 + 4t - 11.$$

Find the velocity at  $t = 5$  seconds.

- A.  $934 \frac{\text{ft}}{\text{sec}}$    B.  $649 \frac{\text{ft}}{\text{sec}}$    C.  $979 \frac{\text{ft}}{\text{sec}}$    D.  **$574 \frac{\text{ft}}{\text{sec}}$**    E. None of the above.

4. (5 points) Consider the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x > 3, \\ x + c & \text{if } x \leq 3. \end{cases}$$

Find the value of  $c$  that makes  $f$  a continuous function.

- A.  $c = -1$    B.  $c = 9$    C.  $c = 1$    D.  **$c = 5$**    E. None of the above.

5. (5 points) Find the equation of the line tangent to the graph  $y = x^2 - 2x + 5$  at  $x = 2$ .

A.  $y = 2x - 2$

**B.  $y = 2x + 1$**

C.  $y = 2x + 5$

D.  $y = 5x - 2$

E. None of the above.

6. (5 points) The revenue (\$) from producing  $x$  widgets per day is modeled by

$$R(x) = 7000x - 0.5x^2.$$

Find and interpret the marginal revenue at  $x = 500$ .

- A. When production level is 500 widgets per day, if we increase production level by 1, the revenue will decrease by approximately \$6,500.
  - B. When production level is 6 widgets per day, if we increase production level by 1, the revenue will increase by approximately \$500.
  - C. When production level is 500 widgets per day, if we increase production level by 6, the revenue will increase by approximately \$1,000.
  - D. When production level is 500 widgets per day, if we increase production level by 1, the revenue will increase by approximately \$6,500.**
  - E. None of the above.
7. (5 points) Consider the function  $f(x) = \frac{x - 4}{x^2 + x - 6}$ . Where is  $f$  continuous?
- A. All real numbers except  $x = 2$ ,  $x = -3$ , and  $x = 4$ .
  - B. All real numbers except  $x = 2$ .
  - C. All real numbers except  $x = -3$ .
  - D. All real numbers except  $x = -3$  and  $x = 2$ .**
  - E. None of the above.

8. (5 points) If  $Q(x) = x^2 + 1$ , compute  $\lim_{h \rightarrow 0} \frac{Q(5+h) - Q(5)}{h}$ .
- A.  $2x$    B.  $10 + h$    C. **10**   D.  $\frac{Q(5) + Q(h) - Q(5)}{h}$    E. None of the above.

9. (5 points) Find the horizontal asymptotes, if any, for  $f(x) = \frac{2x^2 - 5x + 11}{9x^2 - 2}$ .
- A.  $y = 0$
  - B.  $x = \frac{\sqrt{2}}{3}$  and  $x = -\frac{\sqrt{2}}{3}$
  - C.  $x = \frac{2}{9}$
  - D.  $y = \frac{2}{9}$**
  - E. None of the above.

10. (5 points) Compute  $\lim_{x \rightarrow \infty} \frac{x+3}{x^2+2x-9}$ .
- A.  $-\frac{1}{3}$    B. **0**   C.  $\infty$    D.  $-\infty$    E. None of the above.
11. (5 points) Solve the inequality  $\frac{x^2-2x+1}{x-3} \leq 0$ .
- A.  $(-\infty, 3) \cup (3, \infty)$   
B.  $(-\infty, -1]$   
C.  $[-1, 3)$   
D.  $(-\infty, -1] \cup (3, \infty)$   
**E. None of the above.**
12. (5 points) Find the average rate of change for the function  $f(x) = x^2 - 3x + 3$  if  $x$  changes from 0 to 4.
- A. 1**   B. 4   C.  $2x - 3$    D. 3   E. None of the above.
13. (5 points) The market research department of a company recommends that the company manufacture and market a new headphone set. After suitable test marketing, the research department presents the following price-demand equation  $p = 100 - 0.0001x$ , where  $x$  is the demand at price  $\$p$  so that the *revenue function* is

$$R(x) = (100 - 0.0001x)x.$$

The financial department provides the *cost function*

$$C(x) = 7000 + 2x.$$

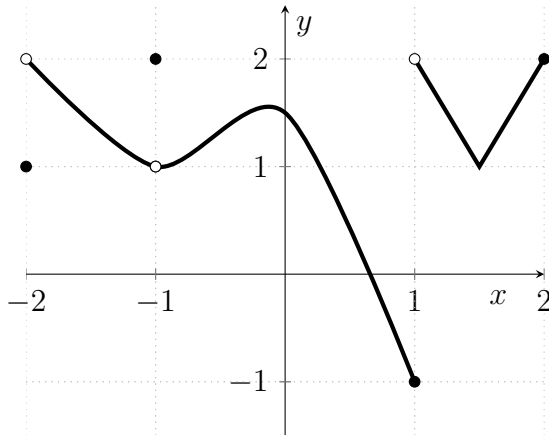
Find and interpret the **marginal profit** at  $x = 6000$  and interpret the results.

- A. At production level 6,000, profits will increase by \$24 per unit increase of production.  
B. At production level 6,000, profits will decrease by \$1,000 per unit increase of production.  
C. At production level 6,000, profits will increase by \$2 per unit increase of production.  
D. At production level 6,000, profits will decrease by \$4 per unit increase of production.  
**E. None of the above.**
14. (5 points) Suppose  $\lim_{x \rightarrow 2} f(x) = 2$  and  $\lim_{x \rightarrow 2} g(x) = -1$ . Compute  $\lim_{x \rightarrow 2} (f(x) + 2g(x))$ .
- A. 0**   B. 1   C. 2   D. 3   E. None of the above.

15. (5 points) Find the point(s) where the graph of  $f(x) = \frac{2}{3}x^3 - 5x^2 + 12x - 1$  has horizontal tangent lines.

- A.  $x = \frac{2}{3}$   
 B.  $x = 0$ .  
 C.  $x = 6$  and  $x = -1$   
**D.  $x = 2$  and  $x = 3$**   
 E. None of the above.

16. (5 points) Below is a graph of  $y = f(x)$ . Find  $\lim_{x \rightarrow 1^+} f(x)$ .

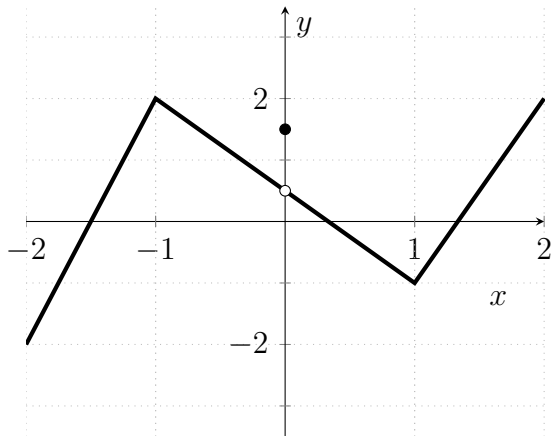


- A. -1    **B. 2**    C. 1    D. -2    E. Does not exist.

17. (5 points) What is the definition of the derivative of  $f(x)$ ?

- A.  $\lim_{h \rightarrow 0} \frac{f(x+h) + f(x)}{h}$   
**B.  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$**   
 C.  $\lim_{h \rightarrow 0} f(x+h) - f(x)$   
 D.  $\lim_{h \rightarrow 0} f(x)$   
 E. None of the above.

18. (5 points) List the  $x$ -values in the graph below at which the function is not differentiable.



- A.  $-1, 0, 1$    B.  $-1, 1$    C.  $-1, \frac{1}{2}, 0$    D.  $1, 0$    E. None of the above.
19. (5 points) Describe the end behavior of  $f(x) = 5x^4 + 5x + 11$ .
- A.  $\lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = \infty$   
 B.  $\lim_{x \rightarrow \infty} f(x) = \infty, \lim_{x \rightarrow -\infty} f(x) = -\infty$   
 C.  $\lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = -\infty$   
 D.  $\lim_{x \rightarrow \infty} f(x) = \infty, \lim_{x \rightarrow -\infty} f(x) = \infty$   
 E. None of the above.
20. (5 points) Find the equation of the tangent line to the curve  $y = f(x)$  at  $x = 9$ , where  $f(x) = -5 - x^2$ .

- A.  $y = -18x + 76$   
 B.  $y = 9x + 76$   
 C.  $y = -18x - 5$   
 D.  $y = -2x$   
 E. None of the above.