Math 120

Final Exam Practice Problems, Form: A

Name: ______________________________

- While every attempt was made to be complete in the types of problems given below, we make no guarantees about the completeness of the problems. Specifically, you are responsible for every section covered in lecture.
- If you find typos/errors in these problems (or solutions) please send an email to `d_yasaki@uncg.edu`.

1. Find the line passing through the points $(1, 1)$ and $(-1, 5)$. Write the equation in standard form.

Solution: To find the equation of a line, we need the slope of the line and a point on the line. The point is given. You can pick $(1, 1)$, for example. To find the slope, we compute rise/run

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - 5}{1 - (-1)} = \frac{-4}{2} = -2.$$

Thus the line is given (in point-slope form) as

$$y - 1 = -2(x - 1).$$

We simplify to find

$$y + 2x = 3.$$

An alternative approach is the following. There is exactly 1 line that goes through the points $(1, 1)$ and $(-1, 5)$. By plugging in and checking, we see that $y + 2x = 3$ is that line. Specifically,

$$1 + 2(1) = 3 \quad \text{and} \quad 5 + 2(-1) = 3.$$

(a) $y - 2x = 3$

(b) $y + 2x = 3$

(c) $y - 2x = -3$

(d) $y + 2x = -3$

(e) None of the above.

2. At a local grocery store the demand for ground beef is approximately 50 pounds per week when the price per pound is \$4, but is only 40 pounds per week when the price rises to \$5.50 per pound. Assuming a linear relationship between the demand x and the price per pound p, express the price as a function of demand.

Solution: We are given that

x	p
50	4.00
40	5.50

Since we are assuming a linear relationship between p and x, this turns into the problem of finding the equation of a line that goes through the points $(50, 4.00)$ and $(40, 5.50)$. We need to find a point on the line and the slope of the line. The point is given. We can take $(50, 4.00)$ for example. To find the slope, we compute rise/run

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{4.00 - 5.50}{50 - 40} = \frac{-1.50}{10} = -0.15.$$

Thus the line is (in point-slope form)

$$p - 4.00 = -0.15(x - 50).$$

We simplify to get

$$p = -0.15x + 11.5.$$

Alternatively, there is exactly 1 line that goes through these 2 points. We can plug in and check. Specifically,

$$4.00 = -0.15(50) + 11.5 \quad \text{and} \quad 5.50 = -0.15(40) + 11.5.$$

(a) $p = -0.15x - 11.5$

(b) $p = 11.5x + 0.15$

(c) $p = 0.15x + 11.5$

(d) $p = 11.5x - 0.15$

(e) $p = -0.15x + 11.5$

3. Compute and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, where $f(x)=5x^2+7x$.

Solution: We compute

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{[5(x+h)^2+7(x+h)]-[5x^2+7x]}{h}\\ &= \frac{[5(x^2+2xh+h^2)+7x+7h]-5x^2-7x}{h}\\ &= \frac{5x^2+10xh+5h^2+7x+7h-5x^2-7x}{h}\\ &= \frac{10xh+5h^2+7h}{h}\\ &= 10x+5h+7.\end{aligned}$$

(a) $15x-7h+14$

(b) $10x^2+5h+7x$

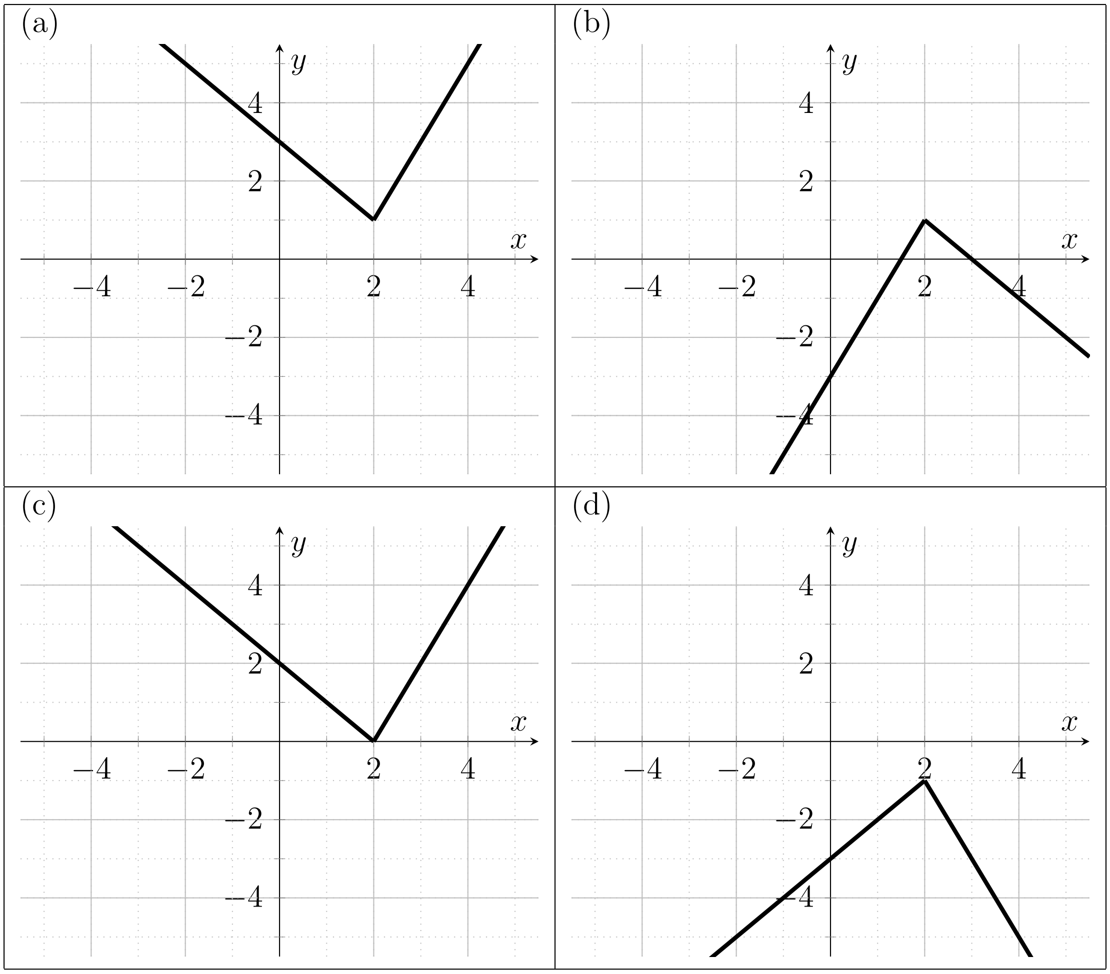
(c) $10x+7$

(d) $10xh+7h$

(e) $10x+5h+7$

4. Graph the function

$$f(x) = \begin{cases} -x+3 & \text{if} \quad x < 2 \\ 2x-3 & \text{if} \quad x \geq 2 \end{cases}$$



Solution: The function f "looks like" $-x+3$ for $x < 2$. This means for $x < 2$, the graph looks like a line with slope -1 and y-intercept 3. The function f "looks like" $2x-3$ for $x \geq 2$. This means that for $x \geq 2$, it is a line with slope 2. Notice that the graph should go through the point $(2, 1)$.

(a) (a)

(b) (b)

(c) (c)

(d) (d)

(e) None of the above.

5. Solve the following quadratic inequality. Express your answer in interval notation.

$$x^2 - 11x + 30 > 0$$

Solution: We need to factor $x^2 - 11x + 30$, and then make a sign chart. By inspection (UNFOIL), or using quadratic formula, we see that

$$x^2 - 11x + 30 = (x - 5)(x - 6).$$

Thus the sign chart is

$$\begin{array}{r|ccccc} x-5: & - & | & + & | & + \\ x-6: & - & | & - & | & + \\ \hline x^2-11x+30: & + & 5 & - & 6 & + \end{array}$$

It follows that $x^2 - 11x + 30 > 0$ on $(-\infty, 5) \cup (6, \infty)$.

(a) $(\infty, 0)$

(b) $(5, 6)$

(c) $(-\infty, 5) \cup (6, \infty)$

(d) $(6, \infty)$

(e) None of the above.

6. Find the vertical asymptote(s) of the graph of the following function:

$$f(x) = \frac{x - 4}{x^2 + 7x}$$

Solution: Recall that the vertical asymptotes of a rational function occur at roots of the denominator (bottom) that are not roots of the numerator (top). We must find the places where the denominator vanishes, and make sure that the numerator does not vanish there. We factor the denominator $x^2 + 7x = x(x + 7)$ and see that the denominator vanishes at $x = 0$ and $x = -7$. We note that $x - 4$ is NOT 0 at either of these values. Thus the vertical asymptotes are $x = 0$ and $x = -7$.

(a) $x = -7, x = 4$

(b) $x = 4, x = 0$

(c) $x = 0$

(d) $x = 0, x = -7$

(e) $x = 0, x = 4, x = -7$

7. Solve for x:

$$2^{4x} = 8^{x+5}$$

Solution: We solve using properties of exponentials.

$$2^{4x} = 8^{x+5} = (2^3)^{x+5} = 2^{3x+15}.$$

It follows that $4x = 3x + 15$. Solving for x, we get $x = 15$.

(a) -15

(b) -5

(c) 15

(d) 5

(e) None of the above.

8. An initial investment of \$12,000 is invested for 2 years in an account that earns 4% interest, compounded quarterly. Find the amount of money in the account at the end of the period.

Solution: Let $A(t)$ denote the amount in the account at time t. We are asked to compute $A(2)$. Recall the general formula for finitely compounded interest

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt},$$

where P is the initial investment, r is the rate as a decimal, n is the number of times compounded per year, and t is the time in years. For this example, we have

$$A(t) = 12000\left(1 + \frac{0.04}{4}\right)^{4t}.$$

We plug in $t = 2$ to find

$$A(2) = 12000\left(1 + \frac{0.04}{4}\right)^{4(2)} \approx 12994.28.$$

(a) \$12,979.20

(b) \$12,994.28

(c) \$994.28

(d) \$12,865.62

(e) None of the above.

9. Solve for x:

$$\ln(3x-4) = \ln(20) - \ln(x-5)$$

Solution: We simplify, using properties of logarithms. Our sub-goal is to get the equation into the form

$$\ln(***) = \ln(@@@).$$

Then solutions to

$$*** = @@@$$

are candidates, and as long as $***$ and @@@ are positive, solutions to this give solutions to the original equation.

Since logarithms can only take positive numbers as input, for our problem any solution we find must satisfy

$$3x-4>0 \quad \text{and} \quad x-5>0.$$

In particular, any solution must satisfy $x > 5$.

$$\begin{aligned} \ln(3x-4) &= \ln(20) - \ln(x-5) \\ &= \ln\left(\frac{20}{x-5}\right) \end{aligned}$$

It follows that we need

$$3x-4 = \frac{20}{x-5}.$$

We solve

$$\begin{aligned} (3x-4)(x-5) &= 20 \\ 3x^2 - 4x - 15x + 20 &= 20 \\ 3x^2 - 19x &= 0 \end{aligned}$$

We see that $x = 0$ and $x = 19/3$ are candidates, but only $19/3$ is greater than 5. Thus the only solution is $19/3$.

(a) 5 and $\frac{5}{3}$

(b) $\frac{19}{3}$ and 0

(c) $\frac{19}{3}$

(d) -5 and $-\frac{19}{3}$

(e) $\frac{29}{4}$

10. If \$4,000 is invested at 7% compounded annually, how long will it take for it to grow to \$6,000, assuming no withdrawals are made? Compute answer to the next higher year if not exact.

Solution: Let $A(t)$ denote the amount in the account after t years. We are asked to find t such that $A(t) = 6000$.

Recall the general formula for finitely compounded interest

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt},$$

where P is the initial investment, r is the rate as a decimal, n is the number of times compounded per year, and t is the time in years. For this example, we have

$$A(t) = 4000(1 + 0.07)^t.$$

We solve

$$\begin{aligned} 4000(1.07)^t &= 6000 \\ (1.07)^t &= 6000/4000 = 6/4 = 3/2 \\ t\ln(1.07) &= \ln(3/2) \\ t &= \frac{\ln(3/2)}{\ln(1.07)} \approx 5.99. \end{aligned}$$

Thus it will take 6 years.

(a) 2 years.

(b) 5 years.

(c) 8 years.

(d) 7 years.

(e) 6 years.

11. Find the following limit, if it exists.

$$\lim_{x \to -4} \frac{x^2 - 16}{x + 4}$$

Solution: If we try to just plug in $x = -4$, we get the indeterminate form. This means we need to DO MORE WORK to compute the limit. This almost always means algebraic simplification.

$$\lim_{x \to -4} \frac{x^2 - 16}{x + 4} = \lim_{x \to -4} \frac{(x-4)(x+4)}{x+4} = \lim_{x \to -4} (x - 4) = -8.$$

(a) ∞

(b) -8

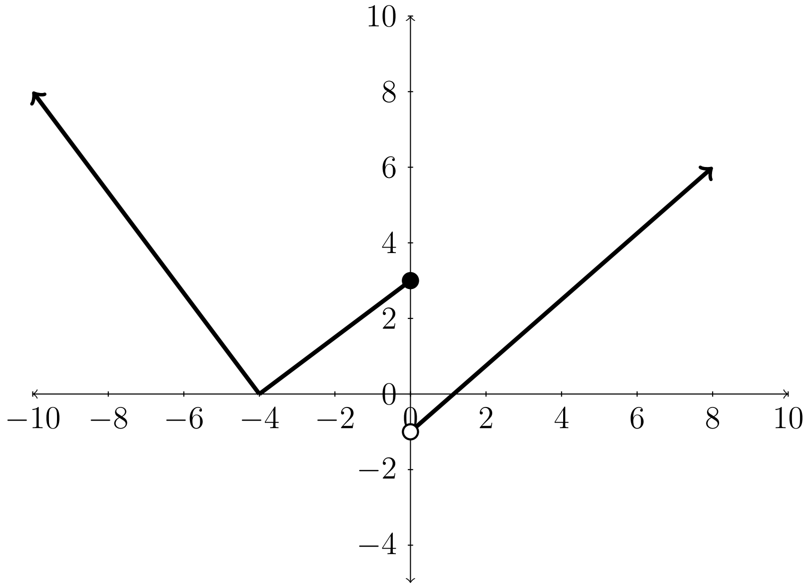
(c) 8

(d) 0

(e) $-\infty$

12. Use the following graph of the function f to evaluate the indicated limits or state that it does not exist.

$$\lim_{x\to 0^-} f(x) \quad \text{and} \quad \lim_{x\to 0^+} f(x)$$



Solution: Recall that $\lim_{x\to 0^-} f(x)$ is asking for the height that is approached by the graph $y = f(x)$ as x approaches 0 from the left. By inspection, we see this is 3. Analogously, $\lim_{x\to 0^+} f(x)$ is asking for the height that is approached by the graph $y = f(x)$ as x approaches 0 from the right. We see this is -1.

(a) $\lim_{x\to 0^-} f(x) = 3$ and $\lim_{x\to 0^+} f(x)$ does not exist.

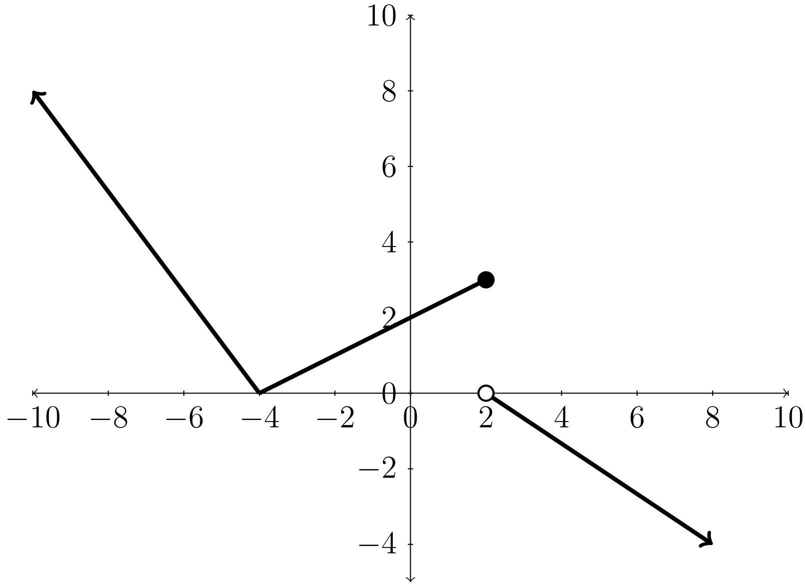
(b) $\lim_{x\to 0^-} f(x) = -1$ and $\lim_{x\to 0^+} f(x) = 3$.

(c) $\lim_{x\to 0^-} f(x)$ does not exist and $\lim_{x\to 0^+} f(x)$ does not exist.

(d) $\lim_{x\to 0^-} f(x) = 3$ and $\lim_{x\to 0^+} f(x) = -1$.

(e) $\lim_{x\to 0^-} f(x)$ does not exist and $\lim_{x\to 0^+} f(x) = -1$.

13. The graph of $f(x)$ is shown. Find all values of x for which f is NOT continuous.



Solution: Recall that a function $f(x)$ is continuous at $x = a$ means

$$\lim_{x \to a} f(x) = f(a).$$

Graphically, this means that $f(x)$ is continuous at $x = a$ as long as the graph $y = f(x)$ does not have a hole or break at $x = a$. We see that $f(x)$ is not continuous at $x = 2$.

(a) 2

(b) -4

(c) 2 and -4

(d) 0 and 3

(e) None of the above.

14. Solve the inequality and express the answer in interval notation:

$$\frac{x^2 - 4x}{x+5} > 0$$

Solution: Recall that a function can only change sign at a zero or a point of discontinuity. Specifically, the candidate points are places where the numerator or the denominator vanish. We factor the numerator as $x^2 - 4x = x(x-4)$ and see that it vanishes when $x = 0, 4$. The denominator vanishes when $x = -5$. We make a sign chart

x :	$-$	|	$-$	|	$+$	|	$+$
$x-4$:	$-$	|	$-$	|	$-$	|	$+$
$x+5$:	$-$	|	$+$	|	$+$	|	$+$
$\frac{x^2-4x}{x+5}$:	$-$	-5	$+$	0	$-$	4	$+$

Thus $\frac{x^2-4x}{x+5} > 0$ on

$$(-\infty, -5) \cup (0, 4).$$

(a) $(-5, 0)$

(b) $(4, \infty)$

(c) $(-5, 0) \cup (4, \infty)$

(d) $(-5, \infty)$

(e) $(-\infty, -5) \cup (0, 4)$

15. Determine the limit.

$$\lim_{x \to 5^+} \frac{x^2}{(x-5)^3}$$

Solution: If we try plugging in $x = 5$, we see that we get $\frac{25}{0}$ (notation, not number). This means that the limit is either ∞ or $-\infty$. To see which one, we can make a sign chart. The only place a sign change can occur is at $x = 0, 5$.

x^2 :	$+$	|	$+$	|	$+$
$(x-5)^3$:	$-$	|	$-$	|	$+$
$\frac{x^2}{(x-5)^3}$:	$-$	0	$-$	5	$+$

It follows that the limit as x approaches 5 from the right is ∞.

(a) 0

(b) ∞

(c) 5

(d) $-\infty$

(e) None of the above

16. Determine the limit.

$$\lim_{x\to\infty} \frac{5x^2+7x-9}{-6x^2+2}$$

Solution: Recall that the long-term behavior of a polynomial is determined by its leading term. In other words, for questions about limits as x goes to ∞, we can replace any polynomial by its leading term. Thus we compute

$$\lim_{x\to\infty} \frac{5x^2+7x-9}{-6x^2+2} = \lim_{x\to\infty} \frac{5x^2}{-6x^2} = \lim_{x\to\infty} \frac{5}{-6} = -\frac{5}{6}.$$

(a) ∞

(b) $-\frac{5}{6}$

(c) $\frac{2}{9}$

(d) 0

(e) $-\infty$

17. Find the equation of the tangent line to the graph of the function at the given value of x.

$$f(x) = x^2 + 5x, \text{ at } x = 4$$

Solution: To compute a tangent line, we need a point on the line and the slope. The point is $(4, f(4))$ and the slope is $f'(4)$. We compute $f(4) = 4^2 + 5(4) = 36$ so the point is $(4, 36)$. We compute $f'(x) = 2x + 5$ so the slope of the tangent line at $x = 4$ is $f'(4) = 2(4) + 5 = 13$. It follows that the line (in point-slope form) is

$$y - 36 = 13(x - 4).$$

We simplify to find the solution in slope-intercept form

$$y = 13x - 16.$$

(a) $y = -\frac{4}{25}x + \frac{8}{5}$

(b) $y = \frac{1}{20}x + \frac{1}{5}$

(c) $y = -39x - 80$

(d) $y = 13x - 16$

(e) None of the above.

18. Find $f'(x)$ if $f(x) = 6x^{-2} + 8x^3 + 11x$.
Solution: We compute

$$\begin{aligned} f'(x) &= 6\frac{d}{dx}(x^{-2}) + 8\frac{d}{dx}(x^3) + 11\frac{d}{dx}(x) \\ 5 &= 6(-2x^{-3}) + 8(3x^2) + 11(1) \\ &= -12x^{-3} + 24x^2 + 11. \end{aligned}$$

(a) $f'(x) = -12x^{-1} + 24x^2 + 11$

(b) $f'(x) = -12x^{-3} + 24x^2 + 11$

(c) $f'(x) = -12x^{-3} + 24x^2$

(d) $f'(x) = -12x^{-1} + 24x^2$

(e) $f'(x) = -12x^{-2} + 24x^3 + 11$

19. An object moves along the y-axis (marked in feet) so that its position at time t (in seconds) is given by $f(t) = 9t^3 - 9t^2 + t + 7$. Find the velocity at one second.
Solution: Recall that velocity is the derivative of the position function. This problem is asking us to compute $f'(1)$. We compute

$$f'(t) = 9(3t^2) - 9(2t) + 1 + 0 = 27t^2 - 18t + 1$$

and so $f'(1) = 27 - 18 + 1 = 10$ feet per second.

(a) 8 feet per second.

(b) 9 feet per second.

(c) 10 feet per second.

(d) 27 feet per second.

(e) 18 feet per second.

20. The demand equation for a certain item is $p = 14 - x/1000$, where x is the demand at price p. The cost equation is $C(x) = 7000 + 4x$. Find the marginal profit at a production level of 3000.
Solution: Let $P(x)$ denote the profit function. The marginal profit is given by $P'(x)$. This problem asks us to compute $P'(3000)$.
Recall that Profit is Revenue minus Cost

$$P(x) = R(x) - C(x).$$

The cost function $C(x)$ is given, and we just need to find the revenue function $R(x)$. Revenue is given by

$$R(x) = p \cdot x = \left(14 - \frac{x}{1000}\right) x = 14x - \frac{x^2}{1000},$$

and so the Profit is

$$\begin{aligned} P(x) &= (14x - \frac{x^2}{1000}) - (7000 + 4x) \\ &= 10x - \frac{x^2}{1000} - 7000. \end{aligned}$$

We compute that

$$P'(x) = 10 - \frac{1}{1000}(2x) - 0 = 10 - \frac{x}{500}$$

and so

$$P'(3000) = 10 - \frac{3000}{500} = 10 - 6 = 4.$$

(a) \$14.

(b) \$4.

(c) \$16.

(d) \$7.

(e) \$11.

21. What will the value of an account (to the nearest cent) be after 8 years if \$100 is invested at 6.0% interest compounded continuously?
Solution: Let $A(t)$ denote the amount in the account at time t. We are asked to compute $A(8)$.
Recall the formula for continuously compounded interest

$$A(t) = Pe^{rt},$$

where P is the initial investment, t is the time in years, and r is the rate as a decimal. For this problem, $A(t) = 100e^{0.06t}$, and so we compute

$$A(8) = 100e^{0.06(8)} \approx 161.60744\ldots$$

(a) \$159.38
(b) \$849.47
(c) \$161.61
(d) \$175.32
(e) \$376.23

22. Find $f'(x)$ for $f(x) = \dfrac{4e^x}{2e^x + 1}$.
Solution: Recall the quotient rule:
Lo D Hi Minus Hi D Lo over Lo Lo
We compute, using the quotient rule

$$\begin{aligned} f'(x) &= \frac{(2e^x+1)(4e^x) - (4e^x)(2e^x)}{(2e^x+1)^2} \\ &= \frac{8e^{2x} + 4e^x - 8e^{2x}}{(2e^x+1)^2} \\ &= \frac{4e^x}{(2e^x+1)^2}. \end{aligned}$$

(a) $f'(x) = \dfrac{4e^x}{(2e^x+1)^2}$
(b) $f'(x) = \dfrac{4e^x}{(2e^x+1)^3}$
(c) $f'(x) = \dfrac{e^x}{(2e^x+1)^2}$
(d) $f'(x) = \dfrac{4e^x}{2e^x+1}$
(e) $f'(x) = \dfrac{8e^{2x}}{(2e^x+1)^2}$

23. Find $f'(x)$ for $f(x) = 4\ln(x^3)$.
Solution: We simplify $f(x)$ first using properties of logarithms

$$f(x) = 4\ln(x^3) = 4(3\ln(x)) = 12\ln(x).$$

We compute

$$f'(x) = \frac{12}{x}.$$

(a) $f(x) = 12\ln(x^3)$
(b) $f(x) = \frac{12}{x^2}$
(c) $f(x) = 12\ln(x^2)$
(d) $f(x) = \frac{4}{x^2}$
(e) $f(x) = \frac{12}{x}$

24. Find $f'(x)$ for $(5x^3 + 4)(3x^7 - 5)$. Do not simplify.
Solution: We compute using the product rule

$$\begin{aligned} f'(x) &= (5x^3 + 4)(3(7x^6) - 0) + (3x^7 - 5)(5(3x^2) + 0) \\ &= (5x^3 + 4)(21x^6) + (3x^7 - 5)(15x^2). \end{aligned}$$

(a) $f'(x) = 15x^2(3x^7 - 5) + (5x^3 + 4)(21x^6)$
(b) $f'(x) = 15x^2(21x^6)$
(c) $f'(x) = 15x^2(3x^7 - 5) - (5x^3 + 4)(21x^6)$
(d) $f'(x) = 15x^2 + 21x^7$
(e) $f'(x) = 15(3x^7 - 5)^2$

25. Find $f'(x)$ for $f(x) = (x^2 + 2)^3$.
Solution: Recall the chain rule and the power rule combine (generalized power rule) to say

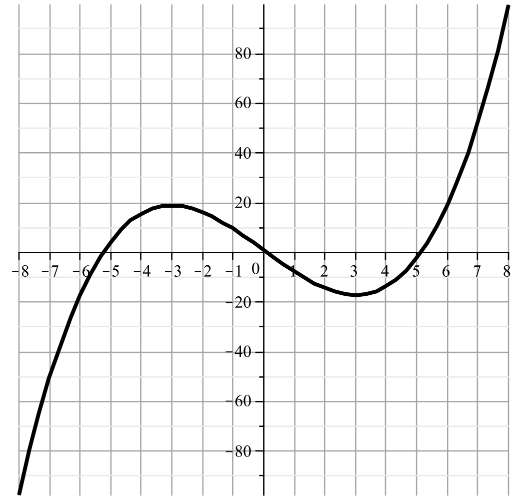
$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}.$$

We use this to compute

$$f'(x) = 3(x^2 + 2)^2(2x).$$

(a) $f'(x) = (2x)^3$
(b) $f'(x) = 3(x^2 + 2)$
(c) $f'(x) = 3(2x + 2)$
(d) $f'(x) = 6x(x^2 + 2)^2$
(e) $f'(x) = (2x)^2$

26. Use the given graph of $f(x)$ to find the intervals on which $f'(x) > 0$.



Solution: Recall that $f'(x) > 0$ means that $f(x)$ is increasing. In other words, the graph $y = f(x)$ goes uphill as you travel to the right. We see that $f(x)$ is increasing on $(-\infty, -3) \cup (3, \infty)$.

(a) $f'(x) > 0$ on $(-\infty, -3) \cup (3, \infty)$.

(b) $f'(x) > 0$ on $(3, -3)$.

(c) $f'(x) > 0$ on $(-\infty, -3)$.

(d) $f'(x) > 0$ on $(-\infty, 3)$.

(e) $f'(x) > 0$ on $(0, \infty)$.

27. Find y'' for $y = -\dfrac{1}{3x+4}$.
Solution: Note that $y = -(3x+4)^{-1}$. We first compute

$$y' = -(-1)(3x+4)^{-2}(3) = 3(3x+4)^{-2}.$$

Then

$$y'' = 3(-2)(3x+4)^{-3}(3) = -18(3x+4)^{-3}.$$

(a) $y'' = -\dfrac{6}{(3x+4)^3}$

(b) $y'' = -\dfrac{2}{(3x+4)^3}$

(c) $y'' = \dfrac{18}{(3x+4)^3}$

(d) $y'' = -\dfrac{18}{(3x+4)^3}$

(e) $y'' = -\dfrac{9}{(3x+4)^3}$

28. Because of material shortages, it is increasingly expensive to produce 6.2L diesel engines. In fact, the profit in millions of dollars from producing x hundred thousand engines is approximated by $P(x) = -x^3 + 30x^2 + 10x - 52$, where $0 \le x \le 20$. Find the inflection point of this function to determine the point of diminishing returns.
Solution: Recall an inflection point is a point at which the function changes concavity. In other words, it is a place at which $P''(x)$ changes sign. We compute

$$P'(x) = -3x^2 + 60x + 10$$

and so

$$P''(x) = -6x + 60.$$

It follows that the inflection point occurs when $x = 10$. The y-coordinate is

$$P(10) = -10^3 + 30(10^2) + 10(10) - 52 = 2048.$$

(a) $(10, 114.67)$

(b) $(10, 1958)$

(c) $(7.50, 2048)$

(d) $(10, 2048)$

(e) None of the above.

29. Where is $f(x) = xe^x$ concave upward?
Solution: Recall that $f(x)$ is concave up where $f''(x) > 0$. We compute (using the product rule)

$$f'(x) = xe^x + e^x.$$

Then

$$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x.$$

We want to see where $f''(x) > 0$. To this end, we factor $xe^x + 2e^x = e^x(x+2)$. We make a sign chart

e^x :	+	|	+
$x+2$:	−	|	+
$xe^x + 2e^x$:	−	−2	+

and see that $f''(x) > 0$ on $(-2, \infty)$.

(a) $(-\infty, -1)$
(b) $(-\infty, -2)$
(c) $(-\infty, 0)$
(d) $(-1, \infty)$
(e) $(-2, \infty)$

30. Determine the local extrema, if any, for the function: $f(x) = x^3 + 3x^2 - 24x + 6$.
Solution: To find local extrema, we use the First Derivative Test. We first compute critical points

$$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x+4)(x-2)$$

and so we have critical points at $x = 2$ and $x = -4$. We make a sign chart for $f'(x)$

$3(x+4)$:	−	|	+	|	+
$x-2$:	−	|	−	|	+
$f'(x)$:	+	−4	−	2	+

It follows that we have a local maximum at $x = -4$ and a local minimum at $x = 2$.

(a) Local max at $x = -4$.
(b) Local max at $x = -4$ and local min at $x = 2$.
(c) Local min at $x = 2$.
(d) Local max at $x = 2$ and local min at $x = -4$.
(e) Local max at $x = 2$

31. Find the absolute maximum and minimum values of $f(x) = 9x^3 - 54x^2 + 81x + 13$ on the interval $[-6, 2]$.
Solution: Recall that to find absolute extrema for continuous function on a closed interval, we need only check the critical points and the endpoints. We compute

$$f'(x) = 27x^2 - 108x + 81 = 27(x^2 - 4x + 3) = 27(x-3)(x-1).$$

Thus we have critical points at $x = 3$ and $x = 1$. However, only $x = 1$ belongs to the interval $[-6, 2]$. We plug this critical point and the endpoints back into $f(x)$ to find the absolute extrema

$$f(-6) = -4361$$
$$f(2) = 31$$
$$f(1) = 49.$$

We see that the absolute minimum is -4361 and the absolute maximum is 49.

(a) $\max f(x) = f(1) = 4361$, $\min f(x) = f(-6) = -49$.
(b) $\max f(x) = f(1) = 49$, $\min f(x) = f(-6) = -4361$.
(c) $\max f(x) = f(1) = 4361$, $\min f(x) = f(-6) = 49$.
(d) $\max f(x) = f(2) = 31$, $\min f(x) = f(-6) = -4361$.
(e) $\max f(x) = f(1) = 49$, $\min f(x) = f(2) = 31$.

32. Find the absolute minimum value of $f(x) = x + \dfrac{9}{x}$ on $(0, \infty)$.
Solution: Note that $f(x) = x + 9x^{-1}$. We compute

$$f'(x) = 1 - 9x^{-2}.$$

The only critical value is $x = 3$. (Note that $x = -3$ is not a critical value since it is not in the domain.) We compute

$$f''(x) = 18x^{-3} \quad \text{and so} \quad f''(3) > 0.$$

Thus by the Second Derivative Test for Absolute Extrema, we have an absolute minimum when $x = 3$. To compute the absolute minimum, we plug back

$$f(3) = 3 + \frac{9}{3} = 6.$$

(a) Absolute minimum is 6 at $x = 3$.
(b) Absolute minimum is 10 at $x = 9$.
(c) Absolute minimum is 3 at $x = 6$.
(d) Absolute minimum is 10 at $x = 1$.
(e) Absolute minimum is 6.5 at $x = 4.5$.

33. A carpenter is building a rectangular room with a fixed perimeter of 360 ft. What are the dimensions of the largest room that can be built?

Solution: Let w be the width and let ℓ be the length. Then the area $A = \ell w$. We are constrained by the perimeter

$$2w + 2\ell = 360.$$

We solve the constraint for ℓ to get

$$\ell = 180 - w.$$

Substitute this in A to get area A as a function of w

$$A(w) = (180 - w)w = 180w - w^2,$$

where w ranges between 0 and 180 (no width to all width). We want to find the maximum of A. We compute

$$A'(w) = 180 - 2w,$$

and so we have a critical point at $w = 90$. Since $A''(w) = -2$, we have that $A''(90) < 0$, and so we have a maximum when $w = 90$. Since $2w + 2\ell = 360$, we find that $\ell = 90$. Thus the largest room that can be built is a square room 90 ft by 90 ft.

(a) 180 ft by 180 ft.

(b) 90 ft by 270 ft.

(c) 90 ft by 90 ft.

(d) 36 ft by 324 ft.

(e) 60 ft by 60 ft.

34. The annual revenue and cost functions for a manufacturer of zip drives are approximately $R(x) = 520x - 0.02x^2$ and $C(x) = 160x + 100,000$, where x denotes the number of drives made. What is the maximum annual profit?
Solution: Recall that the profit is

$$P(x) = R(x) - C(x).$$

We compute

$$P(x) = (520x - 0.02x^2) - (160x + 100000) = 360x - 0.02x^2 - 100000.$$

Then

$$P'(x) = 360 - 0.04x,$$

and $P(x)$ has a critical point at $x = 9000$. Since $P''(x) = -0.04$, we have $P''(9000) < 0$ and so the absolute maximum occurs when $x = 9000$. To find the maximum, we plug back in to $P(x)$

$$P(9000) = 360(9000) - 0.02(9000^2) - 100000 = 1520000.$$

(a) $1,820,000
(b) $1,620,000
(c) $1,520,000
(d) $1,720,000
(e) $1,420,000

35. A computer software company sells 20,000 copies of a certain computer game each year. It costs the company \$1.00 to store each copy of the game for one year. Each time it must produce additional copies, it costs the company \$625 to set up production. Then the storage cost is $S(x) = 0.50x$, and the set up cost is $T(x) = 12500000/x$, where x is the number of games produced in each production run. How many copies of the game should the company produce during each production run in order to minimize its total storage and set-up costs?

Solution: The total cost is

$$C(x) = T(x) + S(x) = 12500000x^{-1} + 0.5x.$$

We compute

$$C'(x) = -12500000x^{-2} + 0.5.$$

Thus we have a critical point at $x = 5000$. Since

$$C''(x) = 25000000x^{-3},$$

we have $C''(5000) > 0$. Then by the Second Derivative Test $C(x)$ has a minimum when $x = 5000$. Since the company wishes to sell 20000 copies, this means we need $20000/5000 = 4$ production runs.

(a) 20,000 copies in 1 production run.

(b) 4000 copies in 5 production runs.

(c) 10,000 copies in 2 production runs.

(d) 5000 copies in 4 production runs.

(e) 9000 copies in 3 production runs.

36. Suppose that \$2200 is invested at 3% interest, compounded semiannually. Find the function for the amount of money after t years.
Solution: We need to use the formula for compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the amount of money after t years, P is the principal (amount at $t = 0$), r is the interest rate as a fraction of 1 and n is the number of compounding periods in a year. For this question, we have $P = 2200$, $r = 0.03$, $n = 2$, hence the function for the amount of money is

$$A = 2200\,(1.015)^{2t}$$

(a) $A = 2200(1.03)^t$
(b) $A = 2200(1.0125)^{2t}$
(c) $A = 2200(1.015)^{2t}$
(d) $A = 2200(1.015)^t$
(e) None of the above.

37. A piece of equipment was purchased by a company for \$10,000 and is assumed to have a salvage value of \$3,000 in 10 years. If its value is depreciated linearly from \$10,000 to \$3,000, find a linear equation in the form $V = mt + b$, where t is the time in years, that will give the salvage value at any time t for $0 \leq t \leq 10$.
Solution: We are given two points on a line and we need to find the equation of that line. The points are $(0, 10000)$ and $(10, 3000)$. The slope m is equal to

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3000 - 10000}{10 - 0} = \frac{-7000}{10} = -700.$$

Using the point $(0, 10000)$ (any point is fine) we obtain the point-slope form of the equation: $V - 10000 = -700(t - 0)$. Simplifying we obtain $V = -700t + 10000$.

(a) $V = -700t - 10000$
(b) $V = 700t - 10000$
(c) $t = -700V - 10000$
(d) $V = -700t + 10000$
(e) $V = 700t + 10000$

38. A couple just had a baby. How much should they invest now at 5.7% compounded daily in order to have $45,000 for the child's education 18 years from now? Compute the answer to the nearest dollar.
Solution: We need to use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ for compound interest. In this case $A = 45000$, $r = 0.057$, $n = 365$, $t = 18$ and we want to find P. Solving for P we obtain

$$P = \frac{A}{(1 + \frac{r}{n})^{nt}} = \frac{45000}{(1 + \frac{0.057}{365})^{365 \cdot 18}} = \frac{45000}{2.78966} = 16130.995 \approx \$16131$$

(a) $16,131
(b) $14,123
(c) $26,530
(d) $45,000
(e) $32,565

39. How would you divide a 16 inch line so that the product of the two lengths is a maximum?
Solution: Let x be the length of one piece of the line. The other piece has length $16 - x$, so we want to maximize the function $p = x(16 - x) = 16x - x^2$. This is a parabola facing down so we know that the maximum occurs at the vertex. The x-coordinate of the vertex is the critical value of the function (there is only one because p' is linear). Solving $p' = 0$ we obtain $16 - 2x = 0$, so $x = 8$. Therefore we need to divide the line segment into two pieces of length 8.

(a) 4 and 12 inches
(b) 8 and 8 inches
(c) 2 and 14 inches
(d) 10 and 6 inches
(e) None of the above.

40. Assume that a savings account earns interest at an annual rate of 2% compounded monthly. How long will it take money invested in this account to double if no withdrawals are made? (round to the nearest year)
Solution: We need to solve for t in the equation $A = P(1 + \frac{r}{n})^{nt}$. We have $r = 0.02$, $n = 12$, $P = 1000$ and the additional condition $A = 2P$. Therefore

$$\begin{aligned} 2P &= P\left(1 + \frac{r}{n}\right)^{nt} \\ 2 &= \left(1 + \frac{r}{n}\right)^{nt} \\ 2 &= \left(1 + \frac{0.02}{12}\right)^{12 \cdot t} \\ 2 &= \frac{601}{600}^{12t} \\ \log 2 &= 12t \cdot \log \frac{601}{600} \\ t &= \frac{\log 2}{12 \log \frac{601}{600}} = 34.68622 \approx 35. \end{aligned}$$

(a) 45 years
(b) 12 years
(c) 35 years
(d) 25 years
(e) 15 years

41. Find the absolute maximum value of the function $f(x) = \dfrac{8}{x^2+1}$.

Solution: First we need the critical values of the function. Since $f(x) = 8(x^2+1)^{-1}$, we obtain

$$f'(x) = -8(x^2+1)^{-2}(2x) = \frac{-16x}{(x^2+1)^2}.$$

Therefore, $f'(x) = 0$ only at $x = 0$.
Using the quotient rule, we obtain the second derivative of f:

$$f''(x) = \frac{(-16)(x^2+1)^2 - (-16x)(2(x^2+1)(2x))}{(x^2+1)^4} = \frac{16(3x^2-1)}{(x^2+1)^3}$$

Hence, $f''(0) = -16 < 0$. Since f has **exactly one** critical value ($x = 0$) and the second derivative is negative at the critical value, we conclude that $f(0) = 8$ is the absolute maximum of the function.
Remark: there is really no need to compute the second derivative. The first derivative $f'(x) = \frac{-16x}{(x^2+1)^2}$ is clearly positive on $(-\infty, 0)$ and negative on $(0, \infty)$. Therefore f increases on $(-\infty, 0)$ to its value 8 at $x = 0$ and then decreases on $(0, \infty)$. It is clear that $f(0) = 8$ is the absolute maximum value taken by f.

(a) 8

(b) -8

(c) 0

(d) 1

(e) -1

42. The financial department of a software design company determined that the cost of producing x palm assistants is $C(x) = 5000 + 3x$. The department also determined the price-demand equation to be $p = 23 - \frac{x}{500}$, where p is the price in dollars. Determine the maximum profit.
Solution: Profit = Revenue - Cost. The Cost function is given, we need to find the Revenue function. But Revenue = price $\times$ demand = $(23 - \frac{x}{500}) \cdot x = 23x - \frac{x^2}{500}$. Therefore, the equation of the profit P is equal to

$$P = R - C = 23x - \frac{x^2}{500} - 5000 - 3x = -\frac{x^2}{500} + 20x - 5000.$$

Since this is the equation of a parabola facing down we know that the maximum value occurs at the vertex (alternatively, P has only one critical value and the second derivative $P'' = -1/250$ is negative).
The derivative $P' = 20 - \frac{x}{250}$ is zero at $x = 5000$, therefore the maximum value of P is $P(5000) = -\frac{5000^2}{500} + 20 \cdot 5000 - 5000 = \$45000$.

(a) $5,000
(b) $25,000
(c) $50,000
(d) $25,00
(e) $45,000

43. Find $f'(x)$ if $f(x) = 9x^{7/5} - 5x^2 + 10000$.
Solution: Using the power rule for derivatives we obtain:

$$f'(x) = (7/5) \cdot 9x^{\frac{7}{5}-1} - 2 \cdot 5x^{2-1} = (63/5)x^{\frac{2}{5}} - 10x.$$

(a) $f'(x) = 63x/5$
(b) $f'(x) = 63x^{2/5}/5$
(c) $f'(x) = \frac{63}{5}x^{2/5} - 10$
(d) $f'(x) = \frac{63}{5}x^{2/5} - 10x$
(e) None of the above.

44. Use the first derivative test to determine the local extrema, if any, for the function $f(x) = x^3 - 3x^2 - 24x + 6$.
Solution: $f'(x) = 3x^2 - 6x - 24 = 3(x^2 - 2x - 8) = 3(x + 2)(x - 4)$.

Sign chart of f':

$$\begin{array}{lrcccccc} \text{Test points:} & f'(-3) & = & 3(-3+2)(-3-4) & = & 21 & > & 0 \\ & f'(0) & = & 3(0+2)(0-4) & = & -24 & < & 0 \\ & f'(5) & = & 3(5+2)(5-4) & = & 21 & > & 0. \end{array}$$

	$(-\infty, -2)$	$(-2, 4)$	$(4, \infty)$
Sign of f'	$+$	$-$	$+$

By the first derivative test, f has a local max at $x = -2$ and a local min at $x = 4$.

(a) Local min at $x = 4$.

(b) Local max at $x = -2$ and local max at $x = 4$.

(c) Local min at $x = -2$.

(d) Local min at $x = 4$ and local max at $x = -2$.

(e) None of the above.

45. Find the standard form of the line with slope $-2/7$ passing through the point $(4, 4)$.
Solution: Since we have a point and the slope, we find first the point-slope form:

$$y - 4 = -\tfrac{2}{7}(x - 4).$$

Hence, $y = -\frac{2}{7}x + \frac{36}{7}$ and the standard form is $\frac{2}{7}x + y = \frac{36}{7}$.

(a) $\frac{2}{7}y + x = 36$

(b) $\frac{36}{7}x + y = \frac{2}{7}$

(c) $\frac{2}{7}x + y = \frac{36}{7}$

(d) $\frac{7}{2}x + y = \frac{7}{36}$

(e) None of the above.

46. Find y'' for $y = \sqrt{5x^2+4}$.
Solution: By the chain rule we obtain

$$y' = \frac{1}{2}\frac{10x}{\sqrt{5x^2+4}} = \frac{5x}{\sqrt{5x^2+4}}.$$

Applying the quotient rule we obtain

$$\begin{aligned} y'' &= \frac{5 \cdot \sqrt{5x^2+4} - 5x \cdot \frac{5x}{\sqrt{5x^2+4}}}{5x^2+4} \\ &= \frac{5(5x^2+4) - 25x^2}{(5x^2+4)^{3/2}} \\ &= \frac{20}{(5x^2+4)^{3/2}}. \end{aligned}$$

(a) $10x$

(b) $5x$

(c) $20/(5x^2+4)^{(3/2)}$

(d) $10x/(5x^2+4)^{(3/2)}$

(e) $5x/\sqrt{5x^2+4}$

47. Find $\lim_{x \to -\infty} \frac{5x^2+3x-1}{3x^4+x^2+1}$.
Solution: The degree of the top is 2. The degree of the bottom is 4. Since the degree of the top is less than the degree of the bottom the limits at ∞ and $-\infty$ are equal to zero.

(a) ∞

(b) $-\infty$

(c) 0

(d) 5/3

(e) The limit does not exist.

48. Determine where the function $H(x) = \dfrac{x^2+7}{x^2+x-6}$ is continuous.

Solution: $H(x) = \dfrac{x^2+7}{x^2+x-6} = \dfrac{x^2+7}{(x+3)(x-2)}$.
Clearly the numerator is never zero. The function is not continuous at the values $x=-3$ and $x=2$ because the denominator is zero at these values and the numerator is not.
Therefore, f is continuous on $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$.

(a) $(2,\infty)$

(b) $x=-3$ and $x=2$.

(c) $(-3,2)$

(d) $(-\infty,-3)\cup(2,\infty)$

(e) $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

49. Find $f'(x)$ for $f(x)=(e^{x^3}+3)^4$.
Solution: By the chain rule,

$$f'(x) = 4(e^{x^3}+3)^3\cdot e^{x^3}\cdot 3x^2 = 12x^2e^{x^3}(e^{x^3}+3)^3.$$

(a) $f'(x)=12x^2(e^{x^3}+3)^3$

(b) $f'(x)=(e^{x^3}+3)^3$

(c) $f'(x)=4x^2e^{x^3}(e^{x^3}+3)^3$

(d) $f'(x)=12x^2e^{x^3}(e^{x^3}+3)^3$

(e) $f'(x)=4e^{x^3}(e^{x^3}+3)^3$

50. Find $\lim\limits_{x\to 3}\dfrac{x+3}{x^2-3x}$.
Solution: As x approaches 3 from the left, the numerator $x+3$ approaches 6 and the denominator approaches 0 from the left, taking (only) negative values arbitrarily close to 0. Hence, $\lim\limits_{x\to 3^-}\dfrac{x+3}{x^2+3x} = -\infty$. Similarly, $\lim\limits_{x\to 3^+}\dfrac{x+3}{x^2+3x}=\infty$.
Since the two one-sided limits are not equal, it follows that $\lim\limits_{x\to 3}\dfrac{x+3}{x^2-3x}$ does not exist.

(a) ∞

(b) $-\infty$

(c) 0

(d) 1

(e) The limit does not exist.

51. Find $\lim_{x \to -1} \dfrac{6x+3}{5x-7}$.
Solution: As x approaches -1, the numerator approaches -3 and the denominator approaches -12. Hence the fraction approaches $-3/-12 = 1/4$, i.e.,

$$\lim_{x \to -1} \frac{6x+3}{5x-7} = \frac{1}{4}.$$

(a) ∞

(b) $-\infty$

(c) 0

(d) 1/4

(e) The limit does not exist.

52. Find the vertical asymptotes of the graph of the function

$$g(x) = \frac{x^2+3x-18}{x^2-2x-15}.$$

Solution:

$$g(x) = \frac{x^2+3x-18}{x^2-2x-15} = \frac{(x+6)(x-3)}{(x-5)(x+3)}.$$

The bottom of the fraction becomes zero for $x = 5$ and $x = -3$. For these values the top is not equal to zero. Therefore the graph of the function has vertical asymptotes $x = 5$ and $x = -3$.

(a) $x = 5$ and $x = -3$

(b) $x = 5$

(c) $x = 3$

(d) There are no vertical asymptotes.

(e) None of the above.

53. Find $\lim_{x \to \infty} \dfrac{-x^3+2x^3-7x-1}{2x^2+5x+11}$.
Solution: The limits at infinity are completely determined by the leading terms:

$$\lim_{x \to \infty} \frac{-x^3+2x^3-7x-1}{2x^2+5x+11} = \lim_{x \to \infty} \frac{-x^3}{2x^2} = \lim_{x \to \infty} \frac{-x}{2} = -\infty.$$

(a) ∞

(b) $-\infty$

(c) 0

(d) $-1/2$

(e) The limit does not exist.

54. A famous painting was sold in 1946 for \$21,770. In 1980 the painting was sold for \$33.2 million. What rate of interest compounded continuously did the investment earn?
Solution: We need to solve for r in $A = Pe^{rt}$, where $A = 33200000$, $P = 21770$ and $t = 1980 - 1946 = 34$:

$$\begin{aligned} 33200000 &= 21770e^{34t} \\ 1525.03445 &= e^{34t} \\ \ln 1525.03445 &= 34t \\ t &= \frac{\ln 1525.03445}{34} = 0.21558 \approx 0.216. \end{aligned}$$

(a) 16%
(b) 26.3%
(c) 21.6%
(d) 12.51%
(e) 5.32%

55. Find the equation of the tangent line to the graph of $f(x) = \ln x^7$ at $x = e^2$.
Solution: Using the properties of logarithms the function is equal to $f(x) = 7\ln x$. Therefore, $f'(x) = 7/x$ so $f'(e^2) = 7/e^2$ which is the slope of the line tangent to the graph at the point $(e^2, f(e^2)) = (e^2, 14)$. The point-slope form of the equation of the line is $y - 14 = (7/e^2)(x - e^2)$. Hence, $y = \frac{7}{e^2}x + 7$.

(a) $y = \frac{e^2}{7}x + 7$
(b) $y = \frac{7}{e^2}x + 7$
(c) $y = \frac{7}{2}x + 7$
(d) $y = \frac{7}{e^2}x - 7$
(e) $y = \frac{7}{e^2}x + e^2$

56. Where does $f(x) = 12e^x - e^{2x}$ have an inflection point?
Solution: $f'(x) = 12e^x - 2e^{2x}$ and $f''(x) = 12e^x - 4e^{2x}$.
Therefore, $f''(x) = 0$ when

$$\begin{aligned} 0 &= 12e^x - 4e^{2x} \\ 12e^x &= 4e^{2x} \\ 12/4 &= e^{2x}/e^x \\ 3 &= e^x \\ \ln 3 &= x. \end{aligned}$$

Sign chart of $f''(x)$:

Test points: $f''(0) = 8 > 0$, $f''(\ln 4) = 48 - 64 = -16 < 0$.

	$(-\infty, \ln 3)$	$(\ln 3, \infty)$
Sign of f''	$+$	$-$

Since the function is defined at $x = \ln 3$ and f'' changes sign at this point, we conclude that f has an inflection point at $x = \ln 3$.

(a) $x = 0$
(b) $x = e^2$
(c) $x = 2\ln 3$
(d) $x = \ln 3$
(e) $x = e^3$

57. Find the derivative of the function $f(x) = 3x^3 \ln x$.
Solution: By the product rule,

$$f'(x) = 9x^2(\ln x) + 3x^3 \cdot \frac{1}{x} = 9x^2(\ln x) + 3x^2 = 3x^2(1 + 3\ln x)$$

(a) $f'(x) = 9x^2(\ln x)^{-1}$
(b) $f'(x) = 9x^2 \ln x$
(c) $f'(x) = 9x^2$
(d) $f'(x) = 3x^2 + 9x^2 \ln x$
(e) $f'(x) = 3x^2 \ln x$

58. Find the local extrema of the following function: $f(x) = 2x^4 - 56x^3 + 13$.
Solution: $f'(x) = 8x^3 - 168x^2 = 8x^2(x - 21)$.
Critical values: $x = 0$, $x = 21$.
Sign chart of f':

Test points: $f'(-1) = 8 \cdot (-22) < 0$; $f'(1) = 8 \cdot (-20) < 0$; $f'(22) = 8 \cdot 22^2 \cdot 1 > 0$.

	$(-\infty, 0)$	$(0, 21)$	$(21, \infty)$
Sign of f'	$-$	$-$	$+$

The function f has a local minimum at $x = 21$ because f' changes from negative to positive at this value (and $f(21)$ is defined). There is no local extremum at $x = 0$ because the sign of f' does not change at this value.

(a) The function has a local maximum at $x = 0$ and a local minimum at $x = 21$.

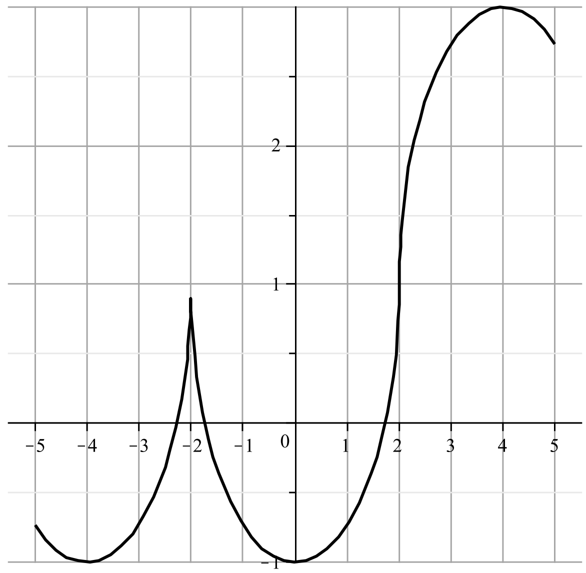
(b) The function has a local maximum at $x = 21$ and a local minimum at $x = 0$.

(c) The function has a local minimum at $x = 0$.

(d) The function has a local minimum at $x = 21$.

(e) The function has no local extrema.

59. Use the following graph of $y = f(x)$ to identify the intervals where $f''(x) > 0$.



Solution: The intervals where $f'' > 0$ correspond to the intervals where f is concave up. The function is concave up on $(-5, -2)$ and on $(-2, 2)$.

(a) $(-5, -4)$, $(-2, 0)$ and $(4, 5)$

(b) $(-5, -2)$ and $(-2, 2)$

(c) $(-5, 2)$

(d) $(2, 5)$

(e) $(-2, 5)$

60. Determine the intervals where the following function is continuous:

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is an integer} \\ -1 & \text{if } x \text{ is not an integer} \end{cases}$$

Solution: The function is discontinuous at every **integer** i since $\lim_{x \to i} f(x) = -1$ is not the value of the function at i. On the other hand if c is **not an integer** then $\lim_{x \to c} f(x) = -1$ is equal to the value of the function at c. Therefore f is continuous at c if and only if c is not an integer. Hence the (maximal) intervals where f is continuous are of the form $(i, i+1)$ for any integer i.

(a) All the intervals of the form $(i, i+1)$, where i is an integer.

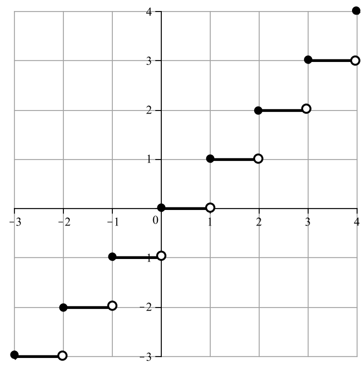
(b) $(-\infty, \infty)$

(c) There are no intervals where the function is continuous.

(d) All the intervals of the form $(i, i+1)$, where i is a real.

(e) $(-1, 1)$

61. The following is the graph of the greatest integer function, which is denoted $[\![x]\!]$ and is defined as $[\![x]\!] =$ greatest integer $\leq x$.



For which c does the limit $\lim_{x \to c^+} [\![x]\!]$ exist?

Solution: By inspection of the graph, it is clear that both $\lim_{x \to c^+} [\![x]\!]$ and $\lim_{x \to c^-} [\![x]\!]$ exist for all real numbers c.

(a) For no c.

(b) Only for integer values of c.

(c) For all real values of c.

(d) For all non-integer values.

(e) None of the above.

62. Find all the asymptotes (horizontal and vertical) of $g(x) = \dfrac{x^2 + 5}{5x^2 - 25}$.

Solution: Horizontal asymptotes: $\lim_{x \to \infty} g(x) = 1/5$ and $\lim_{x \to -\infty} g(x) = 1/5$ because the degree of the top is equal to the degree of the bottom. Hence the line $y = 1/5$ is a horizontal asymptote. Vertical asymptotes: the bottom becomes zero for $x = \sqrt{5}$ and $x = -\sqrt{5}$. Since the top is not zero at these values, it follows that the lines $x = \sqrt{5}$ and $x = -\sqrt{5}$ are vertical asymptotes for the graph of g.

(a) $y = 1/5$ and $x = \sqrt{5}$.

(b) $y = 0$, $x = \sqrt{5}$ and $x = -\sqrt{5}$.

(c) $x = \sqrt{5}$ and $x = -\sqrt{5}$.

(d) $y = 1/5$, $x = \sqrt{5}$ and $x = -\sqrt{5}$.

(e) $y = 0$.

63. Find where the inflection points of $f(x) = \ln(x^2 - 4x + 5)$ occur.
Solution: First note that $x^2 - 4x + 5 > 0$ for all values of x because if we try to find any roots with the formula $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ we get -4 under the square root, so there are no real roots. Therefore the domain of $\ln(x^2 - 4x + 5)$ is $(-\infty, \infty)$.
Using the chain rule we obtain $f'(x)$:

$$f'(x) = \frac{2x - 4}{x^2 - 4x + 5}.$$

Using the quotient rule we obtain $f''(x)$:

$$\begin{aligned} f''(x) &= \frac{2 \cdot (x^2 - 4x + 5) - (2x - 4)(2x - 4)}{(x^2 - 4x + 5)^2} \\ &= \frac{2(x^2 - 4x + 5) - (4x^2 - 16x + 16)}{(x^2 - 4x + 5)^2} \\ &= \frac{2x^2 - 8x + 10 - 4x^2 + 16x - 16}{(x^2 - 4x + 5)^2} \\ &= \frac{-2x^2 + 8x - 6}{(x^2 - 4x + 5)^2} \\ &= \frac{-2(x^2 - 4x + 3)}{(x^2 - 4x + 5)^2} = \frac{-2(x-1)(x-3)}{(x^2 - 4x + 5)^2} \end{aligned}$$

Sign chart of f'':

	$(-\infty, 1)$	$(1, 3)$	$(3, \infty)$
Sign of f''	$-$	$+$	$-$

The sign of f'' changes at $x = 1$ and $x = 3$ (and the function f is defined at those points), therefore f has inflection points at $x = 1$ and at $x = 3$.

(a) $x = 1$

(b) $x = 3$

(c) $x = 1$ and $x = 3$

(d) $x = 2$

(e) There are no inflection points.

64. The fixed costs related to the publication of a book amount to $60,270. The variable costs are equal to $1.60 for each book produced. The book is sold to the distributors for $18 each. How many books should be produced and sold to break even? Round to the nearest whole number.

Solution: Let x be the numbers of books produced (which we assume to be equal to the number of sold books). The cost function is the sum of the fixed and variable costs: $C(x) = 60270 + 1.6x$. The revenue function is simply $R(x) = 18x$. In order to break even we need

$$\begin{aligned} R(x) &= C(x) \\ 18x &= 60270 + 1.6x \\ 16.4x &= 60270 \\ x &= 60270/16.4 = 3675 \text{ books.} \end{aligned}$$

(a) 3075 books

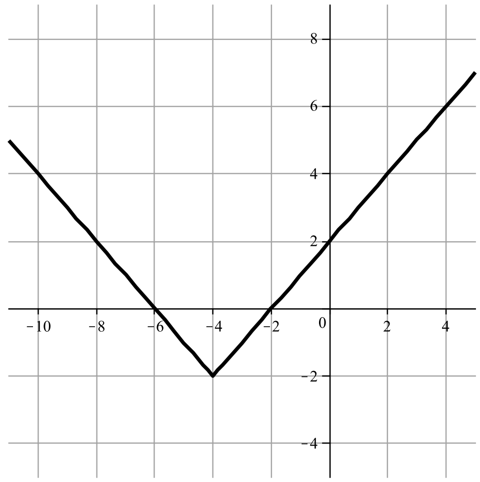
(b) 5357 books

(c) 3675 books

(d) 2093 books

(e) 3348 books

65. Find the equation having the following graph:



Solution: We see that the graph can be obtained by shifting the graph of the absolute value $y = |x|$ four units to the left and two units down. Note that the slope of the two lines in the graph are -1 and 1, which corresponds to the slopes in the graph of the absolute value. Therefore, no vertical stretching was applied. Hence the given graph corresponds to the graph of the function $y = |x+4| - 2$.

(a) $y = |x - 4| - 2$

(b) $y = |x + 2| + 2$

(c) $y = -|x - 4| + 2$

(d) $y = -|x + 4| - 2$

(e) $y = |x + 4| - 2$

66. Solve for x the following equation:

$$2xe^{x^2-1} + x^2e^{x^2-1} = 0$$

Solution:

$$2xe^{x^2-1} + x^2e^{x^2-1} = 0$$
$$x(2+x)e^{x^2-1} = 0$$

The product $x(2+x)e^{x^2-1}$ is zero if and only if one of its factors is zero. But the exponential function is never zero, therefore the zeros of the equation are $x = 0$ and $x = -2$.

(a) $x = 1$, $x = -1$.

(b) There is no solution.

(c) $x = 1$, $x = -1$ and $x = 0$.

(d) $x = -2$, $x = 0$.

(e) None of the above.

67. Find the vertex and the minimum value of the following quadratic polynomial: $f(x) = 2(x-1)^2 + 8$.
Solution: The equation is in vertex form. The minimum value occurs when $x = 1$, which is when the positive term $2(x-1)^2$ vanishes. Hence, the minimum value is $f(1) = 8$ and the vertex is the point $(1, 8)$.

(a) Vertex = $(1, 8)$, minimum value = -8.

(b) Vertex = $(1, -8)$, minimum value = -8.

(c) Vertex = $(1, 8)$, minimum value = 8.

(d) Vertex = $(-1, 8)$, minimum value= 8.

(e) Vertex = $(-1, 8)$, minimum value= -8.

68. Find the vertex of the parabola defined by the equation $f(x) = -2x^2 - 12x - 15$.
Solution: The tangent line to the graph at the vertex is horizontal. Hence, it is enough to solve $f'(x) = 0$ to obtain the x coordinate of the vertex. Since $f'(x) = -4x - 12$, we obtain that $f'(x) = 0$ for $x = -3$. The y coordinate of the vertex is $f(-3) = -2(-3)^2 - 12(-3) - 15 = 3$. Therefore the vertex is the point $(-3, 3)$.

(a) $(-3, 33)$

(b) $(-3, 3)$

(c) $(3, 33)$

(d) $(3, 3)$

(e) $(3, -3)$

69. A mathematical model for the decay of radioactive substances is given by $Q = Q_0e^{rt}$, where Q_0 is the amount of the substance at time $t = 0$, r is the continuous compound rate of decay, t is the time in years and Q is the amount of the substance at time t. If the continuous compound rate of decay of radium per year is $r = -0.0004332$, how long will it take a certain amount of radium to decay to half the original amount (i.e. what is the *half-life* of radium)?

Solution: We have $r = -0.0004332$, $Q = \frac{1}{2}Q_0$ and we want to find t. Therefore,

$$\begin{aligned}
Q &= Q_0e^{rt} \\
(1/2)Q_0 &= Q_0e^{-0.0004332t} \\
1/2 &= e^{-0.0004332t} \\
\ln(1/2) &= -0.0004332t \\
t &= \frac{\ln(1/2)}{-0.0004332} = \frac{\ln 2}{0.0004332} \approx 1600\,\text{years.}
\end{aligned}$$

(a) 1 million years.

(b) 2 years.

(c) 800 years.

(d) 1600 years.

(e) 8 years.

70. Find the vertical asymptotes of the graph of $f(x) = \dfrac{x^2 - 4x + 4}{x^2 - 4}$.

Solution: Clearly,

$$f(x) = \frac{x^2 - 4x + 4}{x^2 - 4} = \frac{(x-2)^2}{(x-2)(x+2)} = \frac{x-2}{x+2}.$$

Then the line $x = -2$ is the only vertical asymptote of the graph of f.

(a) $x = 2$.

(b) $x = 2$ and $x = -2$.

(c) $x = -2$.

(d) $x = 1$

(e) None of the above.

71. Find the values of x where the tangent line to the graph of f is horizontal, if

$$f(x) = \sqrt{x^2 - 12x + 40}.$$

Solution: By the chain rule,

$$f'(x) = \frac{2x - 12}{2\sqrt{x^2 - 12x + 40}} = \frac{x - 6}{\sqrt{x^2 - 12x + 40}}.$$

The tangent line to the graph of f is horizontal when $f'(x) = 0$. The fraction $f'(x) = (x - 6)/\sqrt{x^2 - 12x + 40}$ is zero when the top is zero, so $f'(x) = 0$ for $x = 6$.

(a) $x = 3$

(b) $x = 6$

(c) $x = 9$

(d) $x = 12$

(e) $x = 15$

72. Find the absolute minimum value of $f(x) = x + \dfrac{4}{x}$ on $(0, \infty)$.

Solution: First we compute $f'(x)$ and $f''(x)$:

$$\begin{aligned} f(x) &= x + \frac{4}{x} \\ f'(x) &= 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2} = \frac{(x-2)(x+2)}{x^2} \\ f''(x) &= \frac{8}{x^3} \end{aligned}$$

The only critical value is $x = 2$ because -2 is not in the interval $(0, \infty)$ that we are considering. Since $f''(2) = 1 > 0$ it follows from the second derivative test for absolute extremum that the absolute minimum is attained at $x = 2$ and its value is $f(2) = 4$.

(a) 2

(b) 4

(c) -2

(d) 0

(e) 1

73. Find the local maxima and minima of the function $f(x) = 3x^{5/3} - 20x$.
Solution: Note that the domain of f is $(-\infty, \infty)$. We need to find the critical values of f:

$$f'(x) = 5x^{2/3} - 20 = 5(x^{2/3} - 4) = 5(x^{1/3} - 2)(x^{1/3} + 2)$$

Hence $f'(x) = 0$ if $x^{1/3} = 2$ or $x^{1/3} = -2$. Therefore the critical values of f are $x = 8$ and $x = -8$.
Sign chart of f':

Test points: $f'(-10) > 0$, $f'(0) < 0$, $f'(10) > 0$.

	$(-\infty, -8)$	$(-8, 8)$	$(8, \infty)$
Sign of f'	$+$	$-$	$+$

By the first derivative test, f has a local maximum at $x = -8$ and a local minimum at $x = 8$.

(a) f has a local maximum at $x = -8$ and a local minimum at $x = 8$
(b) f has a local maximum at $x = 8$ and a local minimum at $x = -8$
(c) f has a local maximum at $x = -8$ and a local maximum at $x = 8$
(d) f has a local minimum at $x = -8$ and a local minimum at $x = 8$
(e) None of the above.

74. Find $f'(x)$ for $f(x) = 4^x + \log_4 x$.
Solution: $f'(x) = 4^x(\ln 4) + 1/(x \ln 4)$.

(a) $f'(x) = 4^x(\ln 4) + 1/(x \ln 4)$
(b) $f'(x) = 4^x + 1/(x \ln 4)$
(c) $f'(x) = 4^x(\ln 4) + 1/x$
(d) $f'(x) = 4^x + 1/x$
(e) None of the above.

75. Find $f'(x)$ for $f(x) = \dfrac{3e^x}{1+e^x}$.

Solution:

$$\begin{aligned} f'(x) &= \frac{3e^x(1+e^x) - 3e^x \cdot e^x}{(1+e^x)^2} \\ &= \frac{3e^x}{(1+e^x)^2} \end{aligned}$$

(a) $f'(x) = \dfrac{3e^x}{(1+e^x)^2}$

(b) $f'(x) = \dfrac{3e^x}{(1+e^x)}$

(c) $f'(x) = \dfrac{3e^{2x}}{(1+e^x)^2}$

(d) $f'(x) = \dfrac{e^x}{(1+e^x)^2}$

(e) None of the above.

Answer Key for Exam A

1. (b)
2. (e)
3. (e)
4. (a)
5. (c)
6. (d)
7. (c)
8. (b)
9. (c)
10. (e)
11. (b)
12. (d)
13. (a)
14. (e)
15. (b)
16. (b)
17. (d)
18. (b)
19. (c)
20. (b)
21. (c)
22. (a)
23. (e)
24. (a)
25. (d)
26. (a)
27. (d)
28. (d)
29. (e)
30. (b)
31. (b)
32. (a)
33. (c)
34. (c)
35. (d)
36. (c)
37. (d)
38. (a)
39. (b)
40. (c)
41. (a)
42. (e)
43. (d)
44. (d)
45. (c)
46. (c)
47. (c)
48. (e)
49. (d)
50. (e)
51. (d)
52. (a)
53. (b)
54. (c)
55. (b)
56. (d)
57. (d)
58. (d)
59. (b)
60. (a)
61. (c)
62. (c)
63. (c)
64. (c)
65. (e)
66. (d)
67. (c)
68. (b)
69. (d)
70. (c)
71. (b)
72. (b)
73. (a)
74. (a)
75. (a)