Mini-Lecture 3.3
Properties of Quadratic Functions

Learning Objectives:

1. Graph a quadratic function using transformations
2. Identify the vertex and axis of symmetry of a quadratic function
3. Graph a quadratic function using its vertex, axis, and intercepts
4. Find a quadratic function given its vertex and one other point
5. Find the maximum or minimum value of a quadratic function

Examples:

1. Graph each function by using transformations on the function $f(x) = x^2$.

$(a) f(x) = 2(x-2)^2 - 2 \qquad (b)\ f(x) = -3(x+1)^2 + 3$

2. Find the vertex, axis of symmetry, and intercepts, then graph the function. State the domain and range, where the function is increasing, and where it is decreasing.

$(a) f(x) = 2x^2 - 3x - 2 \qquad (b)\ f(x) = -x^2 - 4x$

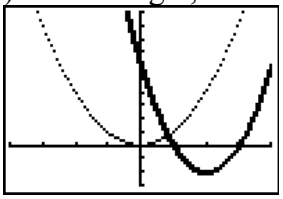
3. Find the quadratic function whose vertex is at (2,5) and passes through (3,2).
4. Determine the value of the maximum or the minimum without graphing.

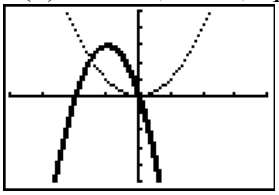
$(a) f(x) = 3x^2 - 24x + 53 \qquad (b)\ f(x) = -2x^2 - 12x - 24$

Teaching Notes:

- Initially, students will get confused with horizontal transformations, but vertical transformations don't cause too much difficulty.
- It is important that they learn to put the function in the form $f(x) = a(x-h)^2 + k$ in order to identify the vertex and to graph the function. It is also a good idea to teach them to use this form to find any *x*-intercepts by solving $a(x-h)^2 + k = 0$, especially when the function does not factor.
- It is important that they use the form $f(x) = ax^2 + bx + c$ to find the *y*-intercept. Otherwise they will think that the *k* value is the *y*-intercept.
- Students will often state the vertex as the maximum or minimum instead of the *y*-value. Emphasize that the maximum or minimum is a value, not a coordinate.

Answers: 1.(a) Move 2 right, down 2, stretch of 2 (b) Reflect, 1 left, up 3, stretch of 3

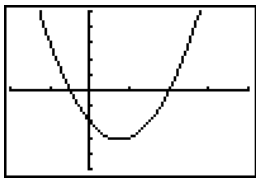




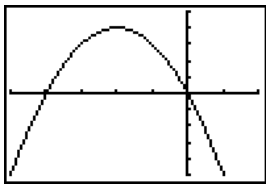
2. (a) Vertex = (3/4,-25/8); Axis of symmetry: x=3/4; *x*-intercepts -1/2, 2; *y*-intercept = -2
Domain = $(-\infty,\infty)$, Range = $[-25/8,\infty)$, Decreasing $(-\infty,3/4)$, Increasing $(3/4,\infty)$

(b) Vertex = (-2,4); Axis of symmetry: x=-2; *x*-intercepts -4, 0; *y*-intercept = 0
Domain = $(-\infty,\infty)$, Range = $(-\infty,4]$, Increasing $(-\infty,-2)$, Decreasing $(-2,\infty)$

(a)



(b)



3. $f(x) = -3(x-2)^2 + 5$

4. (a) Minimum is 5 (b) Maximum is -6