

Mini-Lecture 4.1

Polynomial Functions and Models

Learning Objectives:

1. Identify polynomial functions and their degree
2. Graph polynomial functions using transformations
3. Identify the real zeros of a polynomial function and their multiplicity
4. Analyze the graph of a polynomial function
5. Build cubic models from data

Examples:

1. Determine which functions are polynomials and state the degree. If not, state why.
(a) $f(x) = 5x^3 - 3x + 4$ (b) $f(x) = \sqrt{3}x^2$ (c) $f(x) = \frac{3+x}{x-3}$ (d) $f(x) = \sqrt{x-3}$
2. Form a polynomial whose degree and zeros are given. Don't expand.
(a) Degree 3; zeros: -2, 0, 4 (b) Degree 4; zeros: -2, multiplicity 3; 1, multiplicity 1
3. Find a polynomial function that could form the graph shown below.
4. Use transformations of $y = x^4$ or $y = x^5$ to graph each function.
(a) $f(x) = (x+3)^5$ (b) $f(x) = x^5 + 3$ (c) $f(x) = (x-1)^4 - 2$
5. Sketch the graph of each function by using end-behavior and multiplicity of zeros.
(a) $f(x) = x^3(x-1)(x+3)$ (b) $f(x) = (x+2)^2(x-2)(x+4)$

Teaching Notes:

- It is essential that students understand what constitutes a polynomial function.
- Emphasize the behavior of the functions at their zeros and the end behavior
- Giving students a generic graph using a,b,c instead of numerical values can be helpful in getting them to master the concepts.

Answers:

1. (a) Polynomial, degree 3 (b) Polynomial, degree 2 (c) Not a polynomial because the variable is in the denominator (d) Not a polynomial because of the radical
2. (a) $f(x) = x(x+2)(x-4)$ (b) $f(x) = (x+2)^3(x-1)$
3. $f(x) = x(x+2)(x-3)^2$

