Mini-Lecture 4.2
Properties of Rational Functions

Learning Objectives:
1. Find the domain of a rational function
2. Find the vertical asymptotes of a rational function
3. Find the horizontal or oblique asymptotes of a rational function

Examples:
1. Find the domain of each rational function.

$(a)\ f(x)=\dfrac{2-x}{x+2}$ $(b)\ f(x)=\dfrac{3x^2}{x^2-9}$ $(c)\ f(x)=\dfrac{x-1}{x^2+1}$ $(d)\ f(x)=\dfrac{x}{x^2-4x-5}$

2. Identify vertical asymptotes, horizontal asymptotes, and oblique asymptotes.

$(a)\ f(x)=\dfrac{4x+1}{3-x}$ $(b)\ f(x)=\dfrac{2x^3+3x}{x^2-1}$ $(c)\ f(x)=\dfrac{x^4+1}{x^2-9}$ $(d)\ f(x)=\dfrac{2x}{x^2+1}$

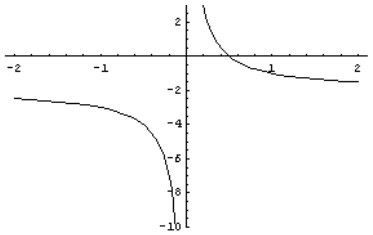
3. Graph each function.

$(a)\ f(x)=\dfrac{1}{x}-2$ $(b)\ f(x)=\dfrac{2}{(x-2)^2}+1$

Teaching Notes:
- Students will often remove from the domain values of x that make the numerator 0. For example, for $f(x)=\dfrac{x-3}{x^2-4}$, they will erroneously state the domain as $\{x|x\neq -2, x\neq 2, x\neq 3\}$.
- Emphasize analyzing the function to determine asymptotes so that the students can learn to determine asymptotes quickly.

Answers:
1. $(a)\ \{x|x\neq -2\}$ $(b)\ \{x|x\neq -3, x\neq 3\}$ (c) All Real Numbers $(d)\ \{x|x\neq -1, x\neq 5\}$

2. (a) V.A. $x=3$, H.A. $y=-4$ (b) Oblique $y=2x$, V.A. $x=1$, $x=-1$ (c) V.A. $x=3$, $x=-3$ (d) H.A. $y=0$

3. (a)  (b)