Mini-Lecture 4.3
The Graph of a Rational Function

Learning Objectives:
1. Analyze the graph of a rational function
2. Solve applied problems involving rational functions

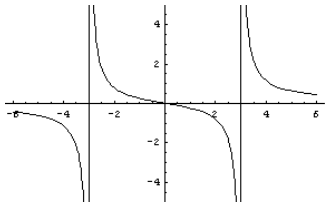
Examples:

1. Analyze the graph of $f(x)=\dfrac{2x}{x^2-9}$ with the 7-step process used in Example 2.
2. A company that produces snowmobiles has a cost function, $C(x)=3500x+150{,}000$.
   (a) Find the average cost function. (b) What is the average cost of producing 100 snowmobiles?

Teaching Notes:

- Students usually do not enjoy application problems, but these problems have a lot of interesting applications. Try to make it relevant for the students by using current data and functions.
- If the students have a graphing calculator, have them try to do the graphs and use the calculator to check their work. They need to grasp the concepts without too much dependency on a graphing calculator to help.
- Emphasize analyzing the functions without looking at the graph. Look at the graphs after the analysis is done, to check your work.

Answers:



1. Step 1: Domain is $\{x \mid x \neq 3, x \neq -3\}$, $f(0)=0$.
   Step 2: Reduced
   Step 3: VA are x = 3 and x = -3
   Step 4: HA is y = 0, intersects at the point (0,0)
   Step 5: On the interval $(-\infty,-3)$, the function is below the *x*-axis.
   On the interval $(-3,0)$, the function is above the *x*-axis.
   On the interval $(0,3)$, the function is below the *x*-axis.
   On the interval $(3,\infty)$, the function is above the *x*-axis.
   Step 6: As $x\to-\infty$, $f(x)\to 0$; $x\to -3^-, f(x)\to-\infty$; $x\to -3^+, f(x)\to\infty$
   $x\to\infty$, $f(x)\to 0$; $x\to 3^-, f(x)\to-\infty$; $x\to 3^+, f(x)\to\infty$
2. (a) $\overline{C}(x)=3500+\dfrac{150{,}000}{x}$ (b) \$5000