Mini-Lecture 4.5
The Real Zeros of a Polynomial Function

Learning Objectives:

1. Use the Remainder and Factor Theorems
2. Use the Rational Zeros Theorem to list the potential rational zeros of a polynomial function
3. Find the real zeros of a polynomial function
4. Solve polynomial equations
5. Use the Theorem for Bounds on Zeros
6. Use the Intermediate Value Theorem

Examples:

1. Find the remainder if $f(x) = x^4 - 3x^3 + 2x - 4$ is divided by $(a)\ x - 5 \quad (b)\ x + 4$
2. Use the Remainder Theorem to determine whether the function $f(x) = 3x^4 - 6x^3 - 11x^2 + 4x + 6$ has the factor $(a)\ (x - 3) \quad (b)\ (x + 2)$.
3. Discuss the real zeros of $f(x) = 5x^5 - 3x^4 + 2x^3 + x^2 - 2x - 5$.
4. For the function $f(x) = 2x^4 + 5x^3 + x^2 + 10x - 6$, (a) list the potential rational zeros, (b) find the rational zeros.
5. Solve the equation $x^4 - 2x^3 - 8x^2 + 10x + 15 = 0$.
6. Find a bound to the zeros of $f(x) = x^5 - 4x^4 + 2x^3 - 5x + 2$.
7. Show that $f(x) = x^4 + x^3 - 9x^2 - 3x + 18$ has a root between -2 and -1.

Teaching Notes:

- This is a very important section for any student that will be taking a calculus course. Make sure the students understand the Division Algorithm for Polynomials since this will form the basis for this section.
- Using a graphing calculator can speed up the process for the Rational Zeros Theorem.
- Emphasize the Intermediate Value Theorem, as this is very important in calculus.

Answers:

1. (a) 256 (b) 436 2. (a) yes (b) no 3. Three or one positive real zeros. Two or none negative real zeros.

4. $(a)\ \frac{p}{q} : \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2} \qquad (b)\ \frac{1}{2}, -3$ 5. $x = 3, -1, \pm\sqrt{5}$ 6. -6 and 6
7. $f(-1) = 12 > 0,\ f(-2) = -4 < 0$