Mini-Lecture 5.1
Composite Functions

Learning Objectives:

1. Form a composite function
2. Find the domain of a composite function

Examples:

1. For $f(x) = 2x + 3$ and $g(x) = x^2 - 2x$, find
   $(a)\ (f \circ g)(2) \quad (b)\ (g \circ f)(2) \quad (c)\ (f \circ f)(-3) \quad (d)\ (g \circ g)(-1)$
2. Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{2}{x-3}$ and $g(x) = \dfrac{1}{x+4}$.
3. Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{1}{x+2}$ and $g(x) = \sqrt{x+1}$.
4. Show that $(f \circ g)(x) = (g \circ f)(x) = x$ for $(f)(x) = 3x - 5$ and $g(x) = \dfrac{x+5}{3}$.
5. Find functions f and g such that $(f \circ g)(x) = H(x)$ for $H(x) = \sqrt{x+5}$.

Teaching Notes:

- The concept of a composite function is not difficult for most students, but the algebraic manipulations that are necessary will be problematic to some.
- Students will struggle a little bit with the domain of a composite function if the functions are rational or radical functions. This will require some time and careful examples.
- Demonstrate that generally $(f \circ g)(x) \neq (g \circ f)(x)$, but that there are cases where $(f \circ g)(x) = (g \circ f)(x) = x$.. Tell them that when this is true there is a special relationship between the two functions that will be studied in future sections.
- If time permits, it would be good to go over the calculus application in Example 6, which is demonstrated in example 5 above. Emphasize the point that there is not a unique answer.

Answers:

1. $(a)\ 3 \quad (b)\ 35 \quad (c)\ -3 \quad (d)\ 3$
2. $\left\{x \mid x \neq -4, x \neq -\dfrac{11}{3}\right\}$
3. $\{x \mid x \geq -1\}$
4. $(f \circ g)(x) = (g \circ f)(x) = x.$
5. Answers can vary. One solution is $f(x) = \sqrt{x}$, $g(x) = x + 5$.