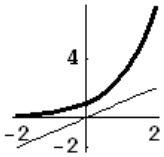
Mini-Lecture 5.2
One-to-One Functions; Inverse Functions

Learning Objectives:

1. Determine whether a function is one-to-one
2. Determine the inverse of a function defined by a map or a set of ordered pairs
3. Obtain the graph of the inverse function from the graph of the function
4. Find the inverse of a function defined by an equation.

Examples:

1. Determine whether the function $\{(2,-1),(3,2),(1,-1),(5,1)\}$ is one-to-one.
2. Find the inverse of the one-to-one function $\{(2,-1),(3,2),(1,3),(5,0)\}$.
3. For the graph shown, draw the graph of the inverse. The graph $y=x$ is included to help.



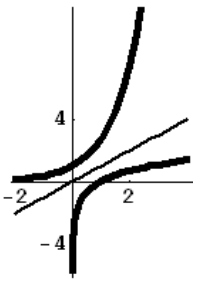
4. For the function $f(x)=x^2-2,\ x\geq 0$, find f^{-1}, state the domain and range of both functions, and graph the functions as well as the line $y=x$.
5. Find the inverse of the function $f(x)=\dfrac{x}{2x+1}$.

Teaching Notes:

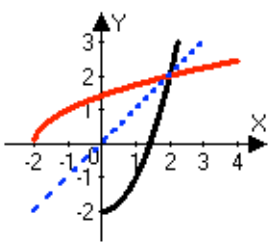
- One of the most common mistakes made is that students think $f^{-1}(x)=\dfrac{1}{f(x)}$. The fact that the -1 is a notation and not an exponent must be made clear.
- It is important that students understand $D_f = R_{f^{-1}}$ and $D_{f^{-1}} = R_f$. This can be demonstrated graphically for emphasis.

Answers:

1. No 2. $\{(-1,2),(2,3),(3,1),(0,5)\}$ 3.



4. $f^{-1}(x)=\sqrt{x+2}$; $D_f = R_{f^{-1}} = [0,\infty)$, $D_{f^{-1}} = R_f = [-2,\infty)$



5. $f^{-1}(x)=\dfrac{x}{1-2x}$