Name:_____

MATH 150: QUIZ 8 (3.3–3.4)

1. Consider the function $f(x) = -3(x+1)^2 + 3$.
   (a) Does the graph of f open up or down?

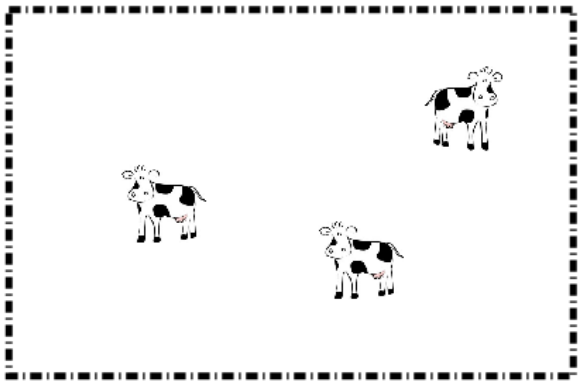
   (b) Find the vertex.

   (c) Find the axis of symmetry.

   (d) Find all of the intercepts of f.

2. Complete the square to write $f(x) = 2x^2 - 3x - 2$ in vertex form.

3. Farmer Brown has 280 yards of fencing. Help him enclose the largest rectangular area possible.



The largest pen possible is a [ ] yd × [ ] yd pen with area [ ] square yards.

Solutions

1. (a) Since the leading coefficient is negative, the graph opens down.
   (b) $(-1, 3)$
   (c) $x = -1$.
   (d) To find the y-intercept, we plug in $x = 0$ and solve for y. We compute
   $$y = -3(0+1)^2 + 3 = 0,$$
   so the y-intercept is 0. To get the x-intercepts, we solve
   $$\begin{aligned} -3(x+1)^2 + 3 &= 0 \\ -3(x+1)^2 &= -3 \\ (x+1)^2 &= 1 \\ (x+1) &= \pm\sqrt{1} = \pm 1 \\ x &= -2, 0. \end{aligned}$$
   Therefore the intercepts are $(-2, 0)$, and $(0, 0)$.
2. We compute
   $$\begin{aligned} f(x) &= 2x^2 - 3x - 2 \\ &= 2\left(x^2 - \frac{3}{2}x\right) - 2 \\ &= 2\left(\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16}\right) - 2 \\ &= 2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{8} - 2 \\ &= 2\left(x - \frac{3}{4}\right)^2 - \frac{25}{8}. \end{aligned}$$
3. Let x denote the width and let y denote the length. Then we have that $2x + 2y = 280$ so that
   $$y = \frac{280 - 2x}{2} = 140 - x.$$
   The area is
   $$A = xy = x(140 - x) = -x^2 + 140x.$$
   We write this in vertex form
   $$\begin{aligned} A(x) &= -x^2 + 140x \\ &= -(x^2 - 140x) \\ &= -((x^2 - 140x + 4900) - 4900) \\ &= -(x^2 - 140x + 4900) + 4900 \\ &= -(x - 70)^2 + 4900. \end{aligned}$$
   It follows that the vertex is $(70, 4900)$ so that the maximum area is 4900 square yards, which occurs with a 70 yd $\times$ 70 yd pen.