Name:_____

MATH 150: QUIZ 10 (4.1)

1. (4 points) For each of the functions below, determine if the function is a polynomial. If it is a polynomial, state the degree.
   (a) $f(x) = 5x^3 - 3x + 4$

   Polynomial Not a polynomial degree = $\square$.

   (b) $f(x) = \sqrt{3}x^2$

   Polynomial Not a polynomial degree = $\square$.

   (c) $f(x) = \dfrac{3+x}{x-3}$

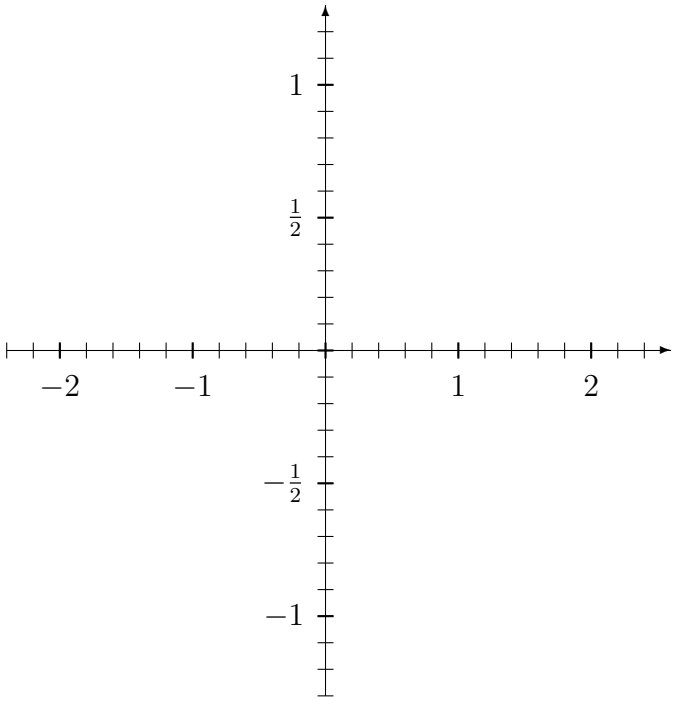
   Polynomial Not a polynomial degree = $\square$.

   (d) $f(x) = \sqrt{x-3}$

   Polynomial Not a polynomial degree = $\square$.

2. (4 points) Form a polynomial with the following properties. Do not expand.
   - Degree 4
   - -2 is a root of multiplicity 3
   - 3 is a root of multiplicity 1
   - y-intercept is 1

3. (4 points) Sketch a graph of the function $f(x) = x^3(x-1)^2(x-2)$ using end-behavior and multiplicity of zeros.

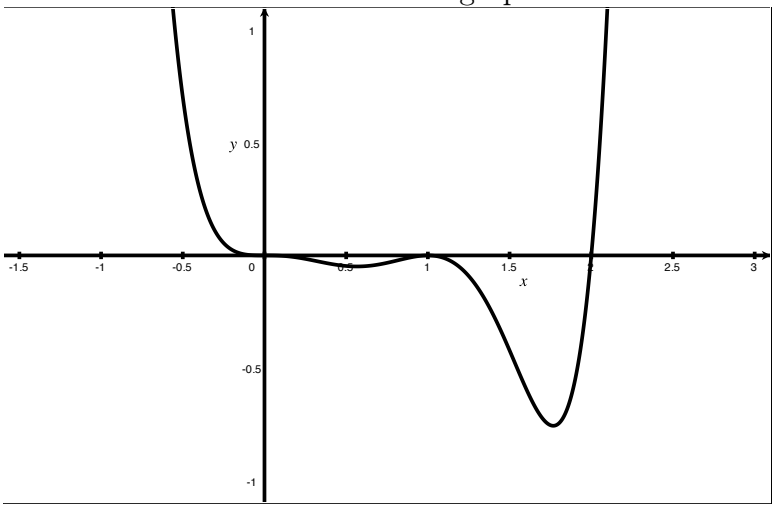


4. (4 points) Let $f(x) = x^4 - 5x^3 + 7x^2 - 4x + 1$.
   (a) Verify that 1 is a root of f.

   (b) Use (a) to factor f into a product of a degree 1 polynomial and a degree 3 polynomial.

Solutions

1. (a) Polynomial of degree 3.
   (b) Polynomial of degree 2.
   (c) Not a polynomial.
   (d) Not a polynomial.
2. The information about the roots and total degree tell us that the polynomial has the form
$$f(x) = a(x+2)^3(x-3)$$
for some value a. The y-intercept tells us that
$$1 = f(0) = a(0+2)^3(0-3) = -24a.$$
It follows that $a = -1/24$ so that
$$f(x) = -\frac{1}{24}(x+2)^3(x-3).$$
3. The key points are listed below.
   - As $x \to \infty$, $f(x) \to \infty$.
   - As $x \to -\infty$, $f(x) \to \infty$.
   - There is a root at 0 and the graph should cross the x-axis there.
   - There is a root at 1 and the graph should not cross the x-axis there.
   - There is a root at 2 and the graph should cross the x-axis there.



4. (a) This can be checked by showing $f(1) = 0$. Alternatively, you can use synthetic division to quickly check that $x - 1$ is a factor of $f(x)$.
   (b) By long division (or synthetic division), we get $(x-1)(x^3 - 4x^2 + 3x - 1)$.