Math 150: Summer 2011

Test 1, Form: A

Name: ______________________

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the questions.
- There are 15 multiple choice and 5 short answer problems on this test. It is your responsibility to make sure that you have all of the problems.
- Each problem is worth 5 points. There is no partial credit given on the multiple choice problems.
- There is no need to complete the test in order. The problems are independent.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

1. Let

$$\begin{aligned} U &= \{\text{red, orange, yellow, green, blue, indigo, violet}\} \\ A &= \{\text{red, orange, yellow, green}\} \\ B &= \{\text{red, orange, indigo, violet}\}. \end{aligned}$$

Compute $A \cup B$.

(a) $\{\text{red, orange}\}$

(b) $\{\text{red, orange, yellow, green, indigo, violet}\}$

(c) $\{\text{blue}\}$

(d) $\emptyset$

(e) None of the above.

2. Simplify the expression $\frac{x^5y^{-6}}{(xy)^2}$. Express the answer so that all the exponents are positive.

(a) $\frac{x^3}{y^8}$

(b) $\frac{x^7}{y^4}$

(c) x^3y^4

(d) $\frac{x^4}{y^8}$

(e) None of the above.

3. Which of the following is equal to $a^3 \cdot a^5$.

(a) a^8

(b) a^{15}

(c) $(a^3)^5$

(d) $\frac{1}{a^2}$

(e) None of the above.

4. Find the distance between the points $(-3, 7)$ and $(4, 10)$.

   (a) 58

   (b) $\frac{3}{7}$

   (c) $\left(\frac{7}{2}, \frac{3}{2}\right)$

   (d) $\sqrt{58}$

   (e) None of the above.

5. Find the midpoint of the line segment joining the points $(6, 3)$ and $(4, 2)$.

   (a) $\frac{1}{2}$

   (b) $(5, 2)$

   (c) $(10, 5)$

   (d) $\left(5, \frac{5}{2}\right)$

   (e) None of the above.

6. In which quadrant does the point $(-3, -2)$ lie?

   (a) I

   (b) II

   (c) III

   (d) IV

   (e) None of the above.

7. Which of the following points are on the graph of the equation

$$y = x^3 - 2x + 3?$$

   (a) $(2, 7)$

   (b) $(7, 2)$

   (c) $(3, 0)$

   (d) $(1, 3)$

   (e) None of the above.

8. Find all the intercepts of

$$y^2 - x - 4 = 0.$$

(a) $(0, 4)$
(b) $(0, 2),\ (0, -2)$
(c) $(1, 3),\ (-1, 3)$
(d) $(0, 2),\ (0, -2),\ (-4, 0)$
(e) None of the above.

9. Which of the following best describes the graph of the equation

$$xy = 1.$$

(a) The graph is symmetric about the x-axis.
(b) The graph is symmetric about the y-axis.
(c) The graph is symmetric about the origin.
(d) The graph is symmetric about the x-axis AND symmetric about the y-axis.
(e) None of the above.

10. Find the intercepts of the line

$$-2x + y = 4.$$

(a) $(0, 4),\ (-2, 0)$
(b) $(4, 0),\ (0, -2)$
(c) $(2, 4),\ (4, 2)$
(d) 2
(e) None of the above.

11. Determine the slope of the line containing the points $(-5, 4)$ and $(0, 7)$.

(a) $\frac{5}{3}$
(b) $\frac{3}{5}$
(c) $\sqrt{34}$
(d) $-\frac{3}{5}$
(e) None of the above.

12. Find the slope and y-intercept of the line

$$y = 3x - 2.$$

(a) slope $= -2$, y-intercept $= 3$.

(b) slope $= 1$, y-intercept $= 1$.

(c) slope $= 3$, y-intercept $= 2$.

(d) slope $= 3$, y-intercept $= -2$.

(e) None of the above.

13. Determine the standard form for the circle with center $(1, 1)$ and radius 2.

(a) $(x + 1)^2 + (y + 1)^2 = 2$

(b) $(x + 1)^2 + (y + 1)^2 = 4$

(c) $(x - 1)^2 + (y - 1)^2 = 2$

(d) $(x - 1)^2 + (y - 1)^2 = 4$

(e) None of the above.

14. Find the center and radius of the circle

$$x^2 + y^2 - 6x + 2y + 4 = 0.$$

(a) Center $= (3, -1)$, radius $= \sqrt{6}$

(b) Center $= (3, -1)$, radius $= 6$

(c) Center $= (-3, 1)$, radius $= \sqrt{6}$

(d) Center $= (-3, 1)$, radius $= 6$

(e) None of the above.

15. Find the equation of the circle of center $(2, -3)$ that contains the point $(0, 4)$. (Hint: Can you compute the radius from the information given?)

(a) $(x + 2)^2 + (y - 3)^2 = 4$

(b) $(x - 2)^2 + (y + 3)^2 = 1$

(c) $(x - 2)^2 + (y + 3)^2 = 53^2$

(d) $(x - 2)^2 + (y + 3)^2 = 53$

(e) None of the above.

16. Evaluate the expression $\dfrac{2x}{x-y}$ at $x = -2$ and $y = 3$.

17. Given that the point $(6, -3)$ is on the graph of an equation that is symmetric with respect to the origin, what other point is on the graph?

18. The lengths of the legs of a right triangle are 32 cm and 60 cm. Find the length of the hypotenuse. Round to the nearest cm.

19. Let ΔABC be the right triangle with $A = (1, 12)$, $B = (6, 7)$, and $C = (1, 0)$. Find the area of ΔABC.

20. Tell me a joke.

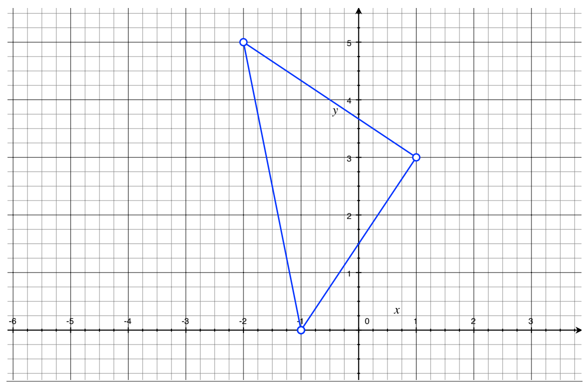
Answer Key for Exam A

1. (b)
2. (a)
3. (a)
4. (d)
5. (d)
6. (c)
7. (a)
8. (d)
9. (c)
10. (a)
11. (b)
12. (d)
13. (d)
14. (a)
15. (d)
16. We plug in $x = -2$ and $y = 3$ into the expression.
$$\frac{2(-2)}{-2-3} = \frac{-4}{-5} = \frac{4}{5}.$$
17. Recall that for a graph to be symmetric about the origin, whenever (a, b) is on the graph, we must have that $(-a, -b)$ is on the graph. It follows that since $(6, -3)$ is on the graph, $(-6, 3)$ is on the graph.
18. By the Pythagorean Theorem, for a right triangle with leg lengths a and b, the hypotenuse length c satisfies
$$a^2 + b^2 = c^2.$$
It follows that
$$\begin{aligned} c^2 &= 32^2 + 60^2 \\ &= 4624 \\ c &= \sqrt{4264} = 68. \end{aligned}$$
The hypotenuse has length 68 cm.

19. There is an error in the problem as stated. Specifically, the problem given is not a right triangle. As a result, I gave everyone credit for this problem. Let me answer a similar problem below.

Let ΔABC be the right triangle with $A = (-2, 5)$, $B = (1, 3)$, and $C = (-1, 0)$. Find the area of ΔABC.

First we sketch the triangle.



Now we compute the lengths of the sides.

$$\begin{aligned} d(B, A) &= \sqrt{|-2-1|^2 + |5-3|^2} \\ &= \sqrt{|-3|^2 + |2|^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13}. \end{aligned}$$

$$\begin{aligned} d(C, A) &= \sqrt{|-2-(-1)|^2 + |5-0|^2} \\ &= \sqrt{|-1|^2 + |5|^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26}. \end{aligned}$$

$$\begin{aligned} d(B, C) &= \sqrt{|-1-1|^2 + |0-3|^2} \\ &= \sqrt{|-2|^2 + |-3|^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13}. \end{aligned}$$

Notice that

$$d(B, A)^2 + d(B, C)^2 = d(C, A)^2,$$

this is a right triangle. Then the area is

$$A = \frac{1}{2}\sqrt{13} \cdot \sqrt{13} = \frac{13}{2}.$$

20. Q: What is brown and sticky?
A: A stick.