Math 150: Summer 2011
Test 3, Form: $\boxed{\text{A}}$

Name: _____

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the questions.

- There are 15 multiple choice and 5 short answer problems on this test. It is your responsibility to make sure that you have all of the problems. There are some BONUS questions at the end.

- Each problem is worth 5 points. There is no partial credit given on the multiple choice problems.

- There is no need to complete the test in order. The problems are independent.

- *Budget your time!*

- If you have read all of these instructions, remember that $1 = 1$.

1. Consider the function $f(x) = -2x + 6$. Compute the average rate of change of f and determine if f increasing, decreasing, or constant.

   (a) 2 and increasing

   (b) -2 and decreasing

   (c) 6 and decreasing

   (d) -6 and decreasing

   (e) None of the above.

2. Consider the data

x	$f(x)$
-1	-2
0	1
1	4
2	7
3	10

   Is the function a linear function? If yes, determine the slope.

   (a) Yes, slope is 3.

   (b) Yes, slope is -3.

   (c) Yes, slope is 1.

   (d) No.

   (e) None of the above.

3. What did one snowman say to the other snowman?

   (a) "The answer to #3 is (d)."

   (b) "I would listen to choice (c)."

   (c) "I agree with (a)."

   (d) "I smell carrots."

   (e) None of the above.

4. Suppose the correlation coefficient of some data x, y is $r = -0.95$. Which of the following best describes the relationship between x and y?

    (a)   When x increases by 1, y decreases by approximately 0.95.

    (b)   There is little or no linear relationship between x and y.

    (c)   There is a strong linear relationship between x and y. As x increases y decreases.

    (d)   There is a strong linear relationship between x and y. As x increases y does also.

    (e)   None of the above.

5. Professor Grant Alexander wanted to find a linear model that relates the number of hours a student plays video games each week to the the cumulative grade point average of the student. The data he collected, with the line of best fit is shown below, where x is the number of hours and y is the grade-point average.
The line of best fit is given by

$$y = -0.0942x + 3.2763.$$

Predict the grade-point-average of a student who plays video games for 8 hours each week, rounded to the nearest hundredth.

    (a)   2.15

    (b)   2.94

    (c)   2.52

    (d)   1.79

    (e)   None of the above.

6. Examine the scatter diagram and determine whether the type of relation is linear or nonlinear.
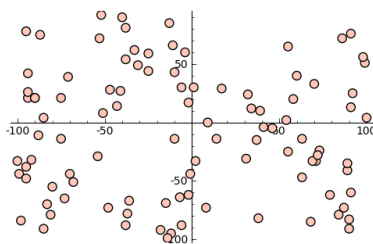
    (a)   Linear

    (b)   Nonlinear

    (c)   Don't pick me!

    (d)   Don't pick me either!

    (e)   None of the above.

7. Find the vertex and axis of symmetry of the graph of

$$f(x) = 2x^2 + 8x + 5.$$

(a) The vertex is $(2, -3)$ and axis of symmetry is $x = 2$.

(b) The vertex is $(0, 5)$ and axis of symmetry is $x = 5$.

(c) The vertex is $(-2, -3)$ and axis of symmetry is $x = -2$.

(d) The vertex is $(-2, 3)$ and axis of symmetry is $x = -2$.

(e) None of the above.

8. Determine the range of

$$f(x) = -3x^2 + 6x + 1$$

(a) $(-\infty, 4]$

(b) $[4, \infty)$

(c) $(-\infty, \infty)$

(d) $(-\infty, 1) \cup (1, \infty)$

(e) None of the above.

9. Suppose the discriminant of the quadratic function g is -4. Which of the following best describes the roots of g?

(a) The roots are 2 and -2.

(b) There is one real root.

(c) There are two distinct real roots.

(d) There are no real roots.

(e) None of the above.

10. The daily revenue R achieved by selling x boxes of candy is figured to be

$$R(x) = 9.5x - 0.04x^2.$$

The daily cost C of selling x boxes of candy is

$$C(x) = 1.25x + 250.$$

The profit P is given by $P(x) = R(x) - C(x)$. How many boxes of candy must the firm sell to maximize profit? Round to the nearest box.

   (a)   103 boxes of candy

   (b)   119 boxes of candy

   (c)   175 boxes of candy

   (d)   564 boxes of candy

   (e)   None of the above.

11. A *Norman window* has the shape of a rectangle surmounted by a semicircle of diameter equal to the width of the rectangle. See figure below. If the perimeter of the window is 20 feet, what dimensions will admit the most light (maximize the area of semicircle and rectangle combined)? Round to the nearest tenth. Hint: For a circle of radius r, the circumference of a circle is $2\pi r$ and the area is πr^2.

$w = 2r$
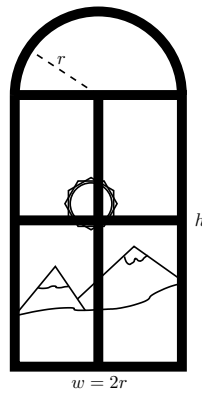
   (a) $w = 2.2$ feet, $h = 7.2$ feet, $r = 1.1$ feet

   (b) $w = 3.4$ feet, $h = 5.6$ feet, $r = 1.7$ feet

   (c) $w = 4.6$ feet, $h = 4.1$ feet, $r = 2.3$ feet

   (d) $w = 5.6$ feet, $h = 2.8$ feet, $r = 2.8$ feet

   (e)   None of the above.

12. The marketing department at Texas Instruments has found that, when certain calculators are sold at a price of p dollars per unit, the number x of calculators sold is given by the demand equation
$$x = 21000 - 150p.$$

Find a model that expresses the revenue R as a function of the price p.

(a) $R(p) = -150p^2 + 21000p$

(b) $R(p) = -150 + \dfrac{21000}{p}$

(c) $R(p) = -150p + 21000$

(d) $R(p) = -\dfrac{x}{150} + 140$

(e) None of the above.

13. Solve the inequality
$$2x^2 < x + 10.$$

(a) $\left(-2, \dfrac{5}{2}\right)$

(b) $(-\infty, -2) \cup \left(\dfrac{5}{2}, \infty\right)$

(c) $(-\infty, 5) \cup (5, \infty)$

(d) $(-\infty, \infty)$

(e) None of the above.

14. Solve the inequality
$$x^2 + x + 1 > 0$$

(a) $\left(-\infty, \dfrac{1 - \sqrt{3}}{2}\right) \cup \left(\dfrac{1 + \sqrt{3}}{2}, \infty\right)$

(b) $\left(\dfrac{1 - \sqrt{3}}{2}, \dfrac{1 + \sqrt{3}}{2}\right)$

(c) $\emptyset$

(d) $(-\infty, \infty)$

(e) None of the above.

15. A ball is thrown vertically upward with an initial velocity of 96 feet per second. The distance s (in feet) of the ball from the ground after t seconds is

$$s(t) = 96t - 16t^2.$$

For what time is the ball more than 128 feet above the ground?

(a)   between 1 and 3 seconds

(b)   between 1.5 and 5 seconds

(c)   between 4 and 6 seconds

(d)   between 2 and 4 seconds

(e)   None of the above.

16. The weekly rental cost of a 20-foot recreational vehicle is $129.50 plus $0.15 per mile.

(a) Find the linear function that expresses cost C as a function of the miles driven x.

$$C(x) = \boxed{}.$$

(b) What is the rental cost if 860 miles are driven?

(c) How many miles were driven if the rental cost is $213.80?

17. A horizontal bridge is in the shape of a parabolic arch. Given the information shown in the figure, what is the height h of the arch 2 feet from the shore?
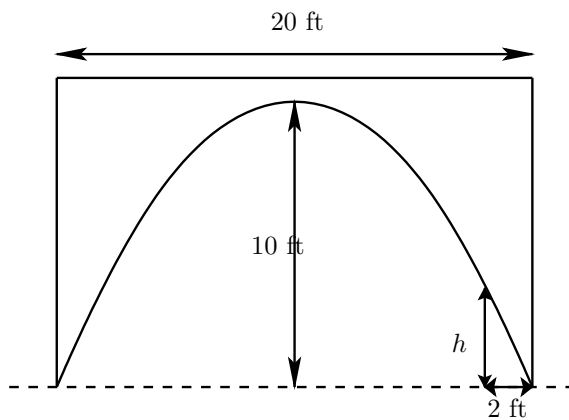
18. The following statement is FALSE. Give a counterexample.

The y-coordinate of the vertex of a parabola gives the minimum of the quadratic function.

19. Find the quadratic function with vertex $(1, 2)$ and y-intercept 5.

20. A pediatrician wants a linear model that relates a child's height H (in inches) to head circumference C (in inches). Let H represent the independent variable and let C represent the dependent variable. After collecting data, the line of best fit is found to be

$$C = 0.3734H + 7.3268.$$

   (a) Explain the significance of the slope 0.3734, or explain why it is not meaningful in this context.

   (b) Explain the significance of the C-intercept 7.3268 or explain why it is not meaningful in this context.

21. BONUS (1 extra point each):

(a) True or False:
$$0.9999\ldots = 1$$

(b) What is the next number in the sequence below? (Hint: This is known as Conway's *look-and-say sequence*)

$$1 \quad 11 \quad 21 \quad 1211 \quad 111221$$

(c) Simplify
$$(x-a)(x-b)(x-c)(x-d)\cdots(x-z).$$

Answer Key for Exam $\boxed{\text{A}}$

1. (b)

2. (a)

3. (d)

4. (c)

5. (c)

6. (b)

7. (c)

8. (a)

9. (d)

10. (a)

11. (d)

12. (a)

13. (a)

14. (d)

15. (d)

16.   (a) $C(x) = 0.15x + 129.50$

    (b) \$258.50

    (c) 562 miles

17. We can put coordinates on the bridge to make things computable. We set the origin at the point on the dotted line below the center of the bridge so that the x-intercepts are 10 and -10 with vertex $(0, 10)$. Then the quadratic equation describing the curve is

$$y = a(x - 10)(x + 10) = ax^2 - 100a,$$

for some value a. Since the vertex is $(0, 10)$, we must have that

$$10 = a(-10)(10) = -100a.$$

It follows that $a = -\dfrac{1}{10}$ so that the quadratic equation is

$$y = -\frac{1}{10}x^2 + 10.$$

To find the height h, note that this corresponds to $x = 8$. We plug in $x = 8$ to find that the height is
$$h = -6.4 + 10 = 3.6 \text{ feet.}$$

18. This statement is only true when the leading coefficient a of the quadratic $ax^2 + bx + c$ is positive. Specifically, the quadratic function $f(x) = -x^2$ is a counterexample.

19. Since we know the vertex, the function must be $f(x) = a(x - 1)^2 + 2$ for some number a. To find a, we note that since the y-intercept is 5, we must have
$$5 = f(0) = a(0 - 1)^2 + 2 = a + 2$$
so that $a = 3$.
$$f(x) = 3(x - 1)^2 + 2$$

20. (a) The units on the slope are inches per inch. The model predicts that for each inch that a child grows, their head circumference grows by 0.3734 inches.

    (b) This would represent the head circumference of a child of 0 height. Nonsense.

21. (a) True. One way to see this is to note that
$$\frac{1}{3} = 0.333\ldots,$$
and then multiply both sides by 3.

    (b) 312211

    (c) 0