# Math 150: Summer 2011

Test 4, Form: A

Name: _____

**Read all of the following information before starting the exam:**

- It is to your advantage to answer ALL of the questions.
- Clearly mark your answers on the Answer Sheet. The Answer Sheet is the ONLY page that will be graded.
- There are 15 multiple choice, 5 true/false, and 4 short answer problems on this test. It is your responsibility to make sure that you have all of the problems. There are some BONUS questions at the end.
- Each multiple choice and short answer problem is worth 5 points. Each true/false is 1 point each.
- There is no need to complete the test in order. The problems are independent.
- *Budget your time!*
- If you have read all of these instructions, remember that $\pi$ is an irrational number that is about 3.
- Tear off the Answer Sheet before beginning your test.

Answer Sheet

## Section 1. Multiple Choice

| | | | | |
|---|---|---|---|---|
| 1. | 4. | 7. | 10. | 13. |
| 2. | 5. | 8. | 11. | 14. |
| 3. | 6. | 9. | 12. | 15. |

## Section 2. True or false

| | | | | |
|---|---|---|---|---|
| True / False | True / False | True / False | True / False | True / False |

## Section 3. Short answer

1. The graph of $f$ [touches / crosses] the $x$-axis at [$-4$ / 6 / 3] since [$-4$ / 6 / 3] is [even / odd / positive / negative].

2. (a) [Polynomial / Not polynomial] degree = ____
   (b) [Polynomial / Not polynomial] degree = ____
   (c) [Polynomial / Not polynomial] degree = ____
   (d) [Polynomial / Not polynomial] degree = ____
   (e) [Polynomial / Not polynomial] degree = ____

3. ____

4. The solutions are $\frac{2}{3}$, ____.

5. Bonus (1 point each):
   (a) ____
   (b) ____
   (c) ____
   (d) [(a) / (b) / (c)]

## Section 1. Multiple choice

1. Suppose the daily cost $C$ of manufacturing bicycles is given by $C(x) = 80x + 5000$. Then the average daily cost $\bar{C}$ is given by $\bar{C}(x) = \dfrac{80x + 5000}{x}$. How many bicycles must be produced each day for the average daily cost to be no more than \$100.

   (a) At least 150 bicycles.

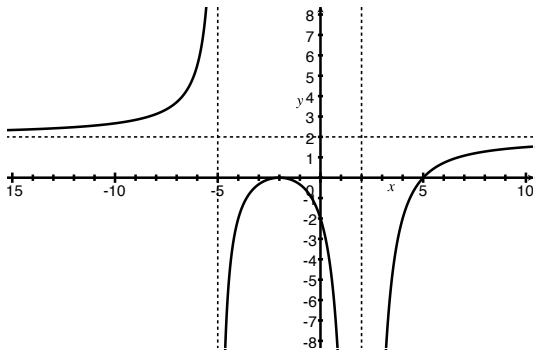
   (b) At least 250 bicycles.

   (c) At least 350 bicycles.

   (d) At least 450 bicycles.

   (e) None of the above.

2. The graph of a rational function $g$ is shown here. The dotted lines indicate asymptotes. Which of the following must be true?




   (a) The numerator of $g$ has a factor of the form $(x-2)^m$, where $m$ is odd.

   (b) The numerator of $g$ has a factor of the form $(x-2)^m$, where $m$ is even.

   (c) The denominator of $g$ has a factor of the form $(x-2)^m$, where $m$ is odd.

   (d) The denominator of $g$ has a factor of the form $(x-2)^m$, where $m$ is even.

   (e) None of the above.

3. Using the *Rational Zeros Theorem*, what are the potential rational zeros of the polynomial $3x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 17$?

   (a) 0.93

   (b) $\pm 1,\ \pm 17,\ \pm \dfrac{1}{3},\ \pm \dfrac{17}{3}$

   (c) $\pm 1,\ \pm 3,\ \pm \dfrac{1}{17},\ \pm \dfrac{3}{17}$

   (d) $\pm 1,\ \pm 17$

   (e) None of the above.

4. Solve $\dfrac{4x+5}{x+2} \geq 3$.

   (a) $(-\infty, -2] \cup [1, \infty)$

   (b) $(-2, 1]$

   (c) $\left[-\dfrac{1}{2}, \infty\right)$

   (d) $(-\infty, -2) \cup [1, \infty)$

   (e) None of the above.

5. Which polynomial has ALL of the following properties?

   - degree 4 polynomial
   - the $y$-intercept is 24
   - $-2$ is a root of multiplicity 3
   - 3 is a root of multiplicity 1

   (a) $(x-24)(x+2)^3(x-3)$

   (b) $-(x+2)^3(x-3)$

   (c) $(x+2)^3(x-3)+24$

   (d) $x^4(x+2)^3(x-3)+24$

   (e) None of the above.

6. My happiness level $H$ (on a scale of 1 to 20) $t$ hours after starting to grade exams is modelled by $H(t) = \dfrac{t^2-2t+17}{t^2+1}$. (I give good grades as long as my happiness is above a 2). Which of the following best describes the time when I am happy enough to grade?

   (a) I am never happy enough to grade.

   (b) Anytime in the first 3 hours.

   (c) Anytime after the first 3 hours.

   (d) Only between hours 2 and 5.

   (e) None of the above.

7. Use the *Bounds on Zeros Theorem* to find a bound on the real zeros of

$$g(x) = 4x^5 - 2x^3 + 2x^2 + 14.$$

(a) Every real zero of $g$ lies between $-\frac{5}{4}$ and $\frac{5}{4}$.

(b) Every real zero of $g$ lies between $-5$ and $5$.

(c) Every real zero of $g$ lies between $-\frac{3}{2}$ and $\frac{3}{2}$.

(d) Every real zero of $g$ lies between $-\frac{1}{2}$ and $\frac{1}{2}$.

(e) None of the above.

8. Which of the following best describes the graph of $y = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$ near $x = 2$.

(a) There is a vertical asymptote $x = 2$.

(b) There is a hole in the graph at $x = 2$.

(c) The graph is above the $x$-axis when $x = 2$..

(d) The graph crosses the $x$-axis at 2.

(e) None of the above.

**For the following three questions, let**

$$G(x) = \frac{3x^2 + x}{x^2 + 2x - 8}.$$

9. Find the domain of $G$.

(a) $\{x \in \mathbb{R} \mid x \neq 2, -4\}$

(b) $\left\{x \in \mathbb{R} \mid x \neq 0, -\frac{1}{3}\right\}$

(c) all real numbers

(d) $\{x \in \mathbb{R} \mid x \neq 0\}$

(e) None of the above.

10. Find the horizontal or oblique asymptotes of the graph of $G$, if any exist.

(a) $y = 3$

(b) $y = 3x + 1$

(c) $x = 2$ and $x = -4$

(d) No horizontal or oblique asymptotes.

(e) None of the above.

11. Find the vertical asymptotes of the graph of $G$, if any exist.

    (a) $y = 3$

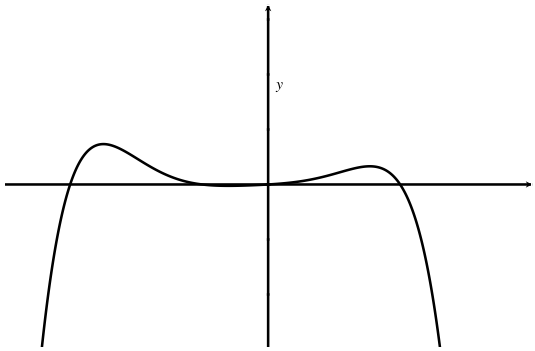
    (b) $x = -2$ and $x = 4$

    (c) $x = 2$ and $x = -4$

    (d) No vertical asymptotes.

    (e) None of the above.

12. Suppose the leading term of a polynomial $f$ is $ax^n$. The graph of $f$ is shown here. Which of the following statements MUST be true?




    (a) $a > 0$ and $n$ is even

    (b) $a > 0$ and $n$ is odd

    (c) $a < 0$ and $n$ is even

    (d) $a < 0$ and $n$ is odd

    (e) Not enough information is given.

13. Find the domain of the rational function

$$R(x) = \frac{3x^2}{x(x-1)(x^2-4)(x^2+1)}.$$

    (a) all real numbers

    (b) $\text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0, 1, 2, -2\}$

    (c) $\text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0\}$

    (d) $\text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0, 1, -1, 2, -2\}$

    (e) None of the above.

14. Find all the rational zeros of the function $f(x) = 2x^3 + 11x^2 - 7x - 6$.

    (a) $1$

    (b) $-6$

    (c) $-3,\ 2,\ 3$

    (d) $-6,\ -\dfrac{1}{2},\ 1$

    (e) None of the above.

15. Give a factorization of $f(x) = x^4 + 2x^3 + 6x^2 - 2x - 7$. Note that 1 and $-1$ are roots of $f$.

    (a) $(x-1)(x+1)(x^2 - x + 3)$

    (b) $(x-1)(x+1)(x^2 + 2x + 2)$

    (c) $(x-1)(x+1)(x^2 + 2x + 7)$

    (d) $(x^2+1)(x^2 - 2x + 11)$

    (e) None of the above.

## Section 2. True or False

_____ $\sqrt{A^2 + B^2} = A + B$

_____ $3a + 2 = 5a$

_____ $\dfrac{n^2}{2n^2 + 1} = \dfrac{1}{2+1}$

_____ $\dfrac{1}{2 + t^{-1}} = \dfrac{1+t}{2}$

_____ $\dfrac{5c^3}{c} = 4c^2$

## Section 3. Short answer

1. Suppose $f$ is a polynomial of degree 6, and $-4$ is a root of $f$ of multiplicity 3. Choose the words that completes the statement most correctly.

   The graph of $f$ | touches | crosses | the $x$-axis at | $-4$ | 6 | 3 | since | $-4$ | 6 | 3 | is | even | odd | positive | negative |.

2. For each of the following, determine if the given function is a polynomial. If it is a polynomial, give its degree.

   (a) $f(x) = 3x^4 + 5x^2 + 1$

(b) $f(x) = \dfrac{x^2 + x + 1}{x - 1}$

(c) $f(x) = \sqrt{x^2 + x - 3}$

(d) $f(x) = x^3 - 2x + \sqrt{3}$

(e) $f(x) = 2x^4 - 7x^{1/2} + 4$

3. Let $f(x) = 4(x^2 + 1)(x - 2)^3$. Find the power function that the graph of $f$ will most resemble for large values of $|x|$.

4. Find all the real solutions of the equation

$$3x^3 + 4x^2 - 7x + 2 = 0.$$

Credit will only be given for exact solutions. (Hint: One of the solutions is $\dfrac{2}{3}$.)

5. BONUS (1 extra point each):

(a) Compute exactly

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}}$$

(b) Compute exactly

$$3 + \cfrac{1}{6 + \cfrac{9}{6 + \cfrac{25}{6 + \cfrac{49}{6 + \cfrac{81}{6 + \cfrac{121}{6 + \cdots}}}}}}$$

(c) Compute $\dfrac{355}{113}$ to 5 decimal places.

(d) Which of the numbers above is $\pi$?

| (a) | (b) | (c) |
|---|---|---|

# Answer Key for Exam A

## Section 1. Multiple choice

| | | | | |
|---|---|---|---|---|
| 1. (b) | 4. (d) | 7. (e) | 10. (a) | 13. (b) |
| 2. (d) | 5. (b) | 8. (b) | 11. (c) | 14. (d) |
| 3. (b) | 6. (b) | 9. (a) | 12. (c) | 15. (c) |

## Section 2. True or False

False $\sqrt{A^2 + B^2} = A + B$

False $3a + 2 = 5a$

False $\dfrac{n^2}{2n^2 + 1} = \dfrac{1}{2 + 1}$

False $\dfrac{1}{2 + t^{-1}} = \dfrac{1 + t}{2}$

False $\dfrac{5c^3}{c} = 4c^2$

## Section 3. Short answer

1. The graph of $f$ [crosses] the $x$-axis at [-4] since [3] is [odd].

2. (a) Polynomial of degree 4.
   (b) Not a polynomial.
   (c) Not a polynomial.
   (d) Polynomial of degree 3.
   (e) Not a polynomial.

3. $y = 4x^5$

4. Long division (or synthetic division) shows that

$$3x^3 + 4x^2 - 7x + 2 = (x - \frac{2}{3})(3x^2 + 6x - 3) = 3(x - \frac{2}{3})(x^2 + 2x - 1).$$

Using quadratic formula or writing the quadratic in vertex form and solving, we see that two additional roots are $-1 + \sqrt{2}$ and $-1 - \sqrt{2}$. The solutions are

$$\frac{2}{3}, \ -1 + \sqrt{2}, -1 - \sqrt{2}.$$

5. BONUS (1 extra point each):

   (a) $\frac{1+\sqrt{5}}{2}$

   (b) $\pi$

   (c) 3.14159

   (d) (b)

# Math 150: Summer 2011

Test 4, Form: B

Name: _____

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Answer Sheet

## Section 1. Multiple Choice

| | | | | |
|---|---|---|---|---|
| 1. | 4. | 7. | 10. | 13. |
| 2. | 5. | 8. | 11. | 14. |
| 3. | 6. | 9. | 12. | 15. |

## Section 2. True or false

| | | | | |
|---|---|---|---|---|
| True / False | True / False | True / False | True / False | True / False |

## Section 3. Short answer

1. The graph of $f$ [touches / crosses] the $x$-axis at [$-4$ / 6 / 3] since [$-4$ / 6 / 3] is [even / odd / positive / negative].

2. (a) [Polynomial / Not polynomial] degree = ____
   (b) [Polynomial / Not polynomial] degree = ____
   (c) [Polynomial / Not polynomial] degree = ____
   (d) [Polynomial / Not polynomial] degree = ____
   (e) [Polynomial / Not polynomial] degree = ____

3. ____

4. The solutions are $\frac{2}{3}$, ____.

5. Bonus (1 point each):
   (a) ____
   (b) ____
   (c) ____
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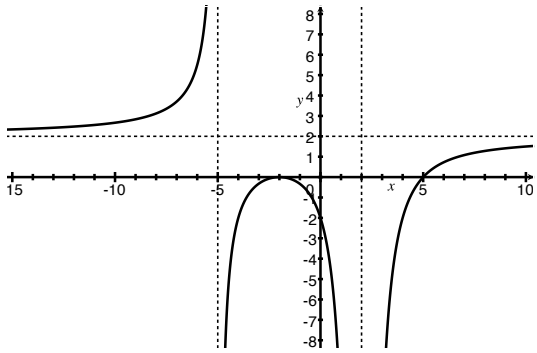
## Section 1. Multiple choice

1. The graph of a rational function $g$ is shown here. The dotted lines indicate asymptotes. Which of the following must be true?

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   (e) None of the above.

2. Give a factorization of $f(x) = x^4 + 2x^3 + 6x^2 - 2x - 7$. Note that 1 and $-1$ are roots of $f$.

   (a) $(x-1)(x+1)(x^2-x+3)$

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   (d) $(x^2+1)(x^2-2x+11)$

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3. Use the *Bounds on Zeros Theorem* to find a bound on the real zeros of

$$g(x) = 4x^5 - 2x^3 + 2x^2 + 14.$$

   (a) Every real zero of $g$ lies between $-\frac{5}{4}$ and $\frac{5}{4}$.

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   (c) Every real zero of $g$ lies between $-\frac{3}{2}$ and $\frac{3}{2}$.

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$$R(x) = \frac{3x^2}{x(x-1)(x^2-4)(x^2+1)}.$$

(a) all real numbers

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5. Suppose the daily cost $C$ of manufacturing bicycles is given by $C(x) = 80x + 5000$. Then the average daily cost $\bar{C}$ is given by $\bar{C}(x) = \dfrac{80x + 5000}{x}$. How many bicycles must be produced each day for the average daily cost to be no more than \$100.

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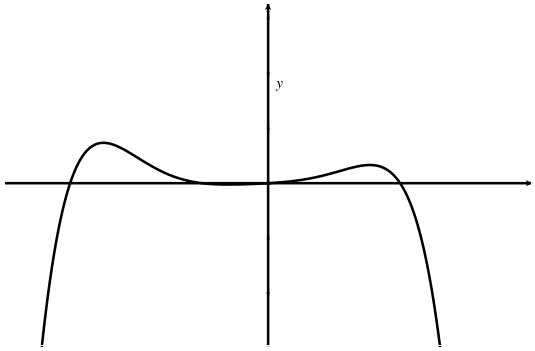


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7. Find all the rational zeros of the function $f(x) = 2x^3 + 11x^2 - 7x - 6$.

   (a) $1$

   (b) $-6$

   (c) $-3,\ 2,\ 3$

   (d) $-6,\ -\dfrac{1}{2},\ 1$

   (e) None of the above.

8. My happiness level $H$ (on a scale of 1 to 20) $t$ hours after starting to grade exams is modelled by $H(t) = \dfrac{t^2 - 2t + 17}{t^2 + 1}$. (I give good grades as long as my happiness is above a 2). Which of the following best describes the time when I am happy enough to grade?

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   (d) Only between hours 2 and 5.

   (e) None of the above.

9. Which polynomial has ALL of the following properties?

   - degree 4 polynomial
   - the $y$-intercept is 24
   - $-2$ is a root of multiplicity 3
   - 3 is a root of multiplicity 1

   (a) $(x - 24)(x + 2)^3(x - 3)$

   (b) $-(x + 2)^3(x - 3)$

   (c) $(x + 2)^3(x - 3) + 24$

   (d) $x^4(x + 2)^3(x - 3) + 24$

   (e) None of the above.

10. Solve $\dfrac{4x + 5}{x + 2} \geq 3$.

    (a) $(-\infty, -2] \cup [1, \infty)$

    (b) $(-2, 1]$

    (c) $\left[-\dfrac{1}{2}, \infty\right)$

    (d) $(-\infty, -2) \cup [1, \infty)$

    (e) None of the above.

11. Using the *Rational Zeros Theorem*, what are the potential rational zeros of the polynomial $3x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 17$?

    (a) 0.93

    (b) $\pm 1,\ \pm 17,\ \pm \dfrac{1}{3},\ \pm \dfrac{17}{3}$

    (c) $\pm 1,\ \pm 3,\ \pm \dfrac{1}{17},\ \pm \dfrac{3}{17}$

    (d) $\pm 1,\ \pm 17$

    (e) None of the above.

12. Which of the following best describes the graph of $y = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$ near $x = 2$.

    (a) There is a vertical asymptote $x = 2$.

    (b) There is a hole in the graph at $x = 2$.

    (c) The graph is above the $x$-axis when $x = 2$..

    (d) The graph crosses the $x$-axis at 2.

    (e) None of the above.

**For the following three questions, let**

$$G(x) = \frac{3x^2 + x}{x^2 + 2x - 8}.$$

13. Find the domain of $G$.

    (a) $\{x \in \mathbb{R} \mid x \neq 2, -4\}$

    (b) $\left\{x \in \mathbb{R} \mid x \neq 0, -\dfrac{1}{3}\right\}$

    (c) all real numbers

    (d) $\{x \in \mathbb{R} \mid x \neq 0\}$

    (e) None of the above.

14. Find the horizontal or oblique asymptotes of the graph of $G$, if any exist.

    (a) $y = 3$

    (b) $y = 3x + 1$

    (c) $x = 2$ and $x = -4$

    (d) No horizontal or oblique asymptotes.

    (e) None of the above.

15. Find the vertical asymptotes of the graph of $G$, if any exist.

    (a) $y = 3$

    (b) $x = -2$ and $x = 4$

    (c) $x = 2$ and $x = -4$

    (d) No vertical asymptotes.

    (e) None of the above.

## Section 2. True or False

_____ $\sqrt{A^2 + B^2} = A + B$

_____ $\dfrac{5c^3}{c} = 4c^2$

_____ $3a + 2 = 5a$

_____ $\dfrac{1}{2 + t^{-1}} = \dfrac{1 + t}{2}$

_____ $\dfrac{n^2}{2n^2 + 1} = \dfrac{1}{2 + 1}$

## Section 3. Short answer

1. Suppose $f$ is a polynomial of degree 6, and $-4$ is a root of $f$ of multiplicity 3. Choose the words that completes the statement most correctly.

   The graph of $f$ | touches | crosses | the $x$-axis at | $-4$ | 6 | 3 | since | $-4$ | 6 | 3 | is | even | odd | positive | negative |.

2. For each of the following, determine if the given function is a polynomial. If it is a polynomial, give its degree.

   (a) $f(x) = 3x^4 + 5x^2 + 1$

   (b) $f(x) = \dfrac{x^2 + x + 1}{x - 1}$

   (c) $f(x) = \sqrt{x^2 + x - 3}$

   (d) $f(x) = x^3 - 2x + \sqrt{3}$

   (e) $f(x) = 2x^4 - 7x^{1/2} + 4$

3. Let $f(x) = 4(x^2 + 1)(x - 2)^3$. Find the power function that the graph of $f$ will most resemble for large values of $|x|$.

4. Find all the real solutions of the equation

$$3x^3 + 4x^2 - 7x + 2 = 0.$$

Credit will only be given for exact solutions. (Hint: One of the solutions is $\frac{2}{3}$.)

5. BONUS (1 extra point each):

(a) Compute exactly

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}}$$

(b) Compute exactly

$$3 + \cfrac{1}{6 + \cfrac{9}{6 + \cfrac{25}{6 + \cfrac{49}{6 + \cfrac{81}{6 + \cfrac{121}{6 + \cdots}}}}}}$$

(c) Compute $\frac{355}{113}$ to 5 decimal places.

(d) Which of the numbers above is $\pi$?

| (a) | (b) | (c) |
|---|---|---|

# Answer Key for Exam B

## Section 1. Multiple choice

| | | | | |
|---|---|---|---|---|
| 1. (d) | 4. (b) | 7. (d) | 10. (d) | 13. (a) |
| 2. (c) | 5. (b) | 8. (b) | 11. (b) | 14. (a) |
| 3. (e) | 6. (c) | 9. (b) | 12. (b) | 15. (c) |

## Section 2. True or False

False $\sqrt{A^2 + B^2} = A + B$

False $\dfrac{5c^3}{c} = 4c^2$

False $3a + 2 = 5a$

False $\dfrac{1}{2 + t^{-1}} = \dfrac{1 + t}{2}$

False $\dfrac{n^2}{2n^2 + 1} = \dfrac{1}{2 + 1}$

## Section 3. Short answer

1. The graph of $f$ [crosses] the $x$-axis at [-4] since [3] is [odd].

2. (a) Polynomial of degree 4.
   (b) Not a polynomial.
   (c) Not a polynomial.
   (d) Polynomial of degree 3.
   (e) Not a polynomial.

3. $y = 4x^5$

4. Long division (or synthetic division) shows that

$$3x^3 + 4x^2 - 7x + 2 = (x - \frac{2}{3})(3x^2 + 6x - 3) = 3(x - \frac{2}{3})(x^2 + 2x - 1).$$

Using quadratic formula or writing the quadratic in vertex form and solving, we see that two additional roots are $-1 + \sqrt{2}$ and $-1 - \sqrt{2}$. The solutions are

$$\frac{2}{3}, \ -1 + \sqrt{2}, -1 - \sqrt{2}.$$

5. BONUS (1 extra point each):

   (a) $\frac{1+\sqrt{5}}{2}$

   (b) $\pi$

   (c) 3.14159

   (d) (b)