

Name: \_\_\_\_\_ Academic Integrity Signature: \_\_\_\_\_

*I have abided by the UNCG Academic Integrity Policy.*

**Note:** Correct numerical answers without justification will receive little or no credit.

1. (3 points) What is  $\frac{d}{dx}(\cos^{-1}(u))$ ?

**Solution:**  $\frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$ , for  $|u| < 1$

2. (3 points) What is  $\frac{d}{dx}(\tan^{-1}(u))$ ?

**Solution:**  $\frac{1}{1+u^2} \cdot \frac{du}{dx}$

3. (4 points) A cylindrical tank of radius 10 feet and height 100 feet is filled with water. How fast is the water level changing if the tank is drained at a constant rate of  $2 \text{ ft}^3/\text{min}$ .

**Solution:** Let  $V$  denote the volume of water, then the height is  $h$ . The problem asks us to compute  $\frac{dh}{dt}$  assuming  $\frac{dV}{dt} = -2$ . Note the minus sign since the volume is getting smaller.

First we relate the volume and height by  $V = 100\pi h$  (since in general the volume of a cylinder of radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .) Differentiate both sides with respect to  $t$ , then plug in the value for  $\frac{dV}{dt}$  and solve for  $\frac{dh}{dt}$ .

$$\begin{aligned} V &= 100\pi h \\ \frac{dV}{dt} &= 100\pi \frac{dh}{dt} \\ -2 &= 100\pi \frac{dh}{dt} \\ \frac{-1}{50\pi} &= \frac{dh}{dt} \end{aligned}$$

Thus the water level is changing at  $\frac{-1}{50\pi} \text{ ft/min}$ . Equivalently water level is falling at  $\frac{1}{50\pi} \text{ ft/min}$ .