MAT 191 Test 2 October 9, 2012

Name: ________________________ Academic Integrity Signature: ________________________

I have abided by the UNCG Academic Integrity Policy.

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the 8 questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- Correct numerical answers with insufficient justification may receive little or no credit.
- Clearly distinguish your final answer from your scratch work with a box or circle.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

Page:	1	2	3	4	5	Total
Points:	32	13	25	14	16	100
Score:						

1. (a) (5 points) If $f(x)$ is a function, give the definition of the *derivative of* $f(x)$.

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) (5 points) Suppose $f(x) = \frac{1}{x}$. **Use the definition** to compute the derivative of $f(x)$.

Solution: We compute

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) \\ &= \lim_{h \to 0} \frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right) \\ &= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{\cancel{h}}{x(x+h)}\right) \\ &= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{-\cancel{h}}{x+h}\right) \\ &= \lim_{h \to 0} \frac{-1}{x(x+h)} \\ &= \frac{-1}{x^2} \end{aligned}$$

2. (10 points) Find the value of a that makes the following function differentiable.

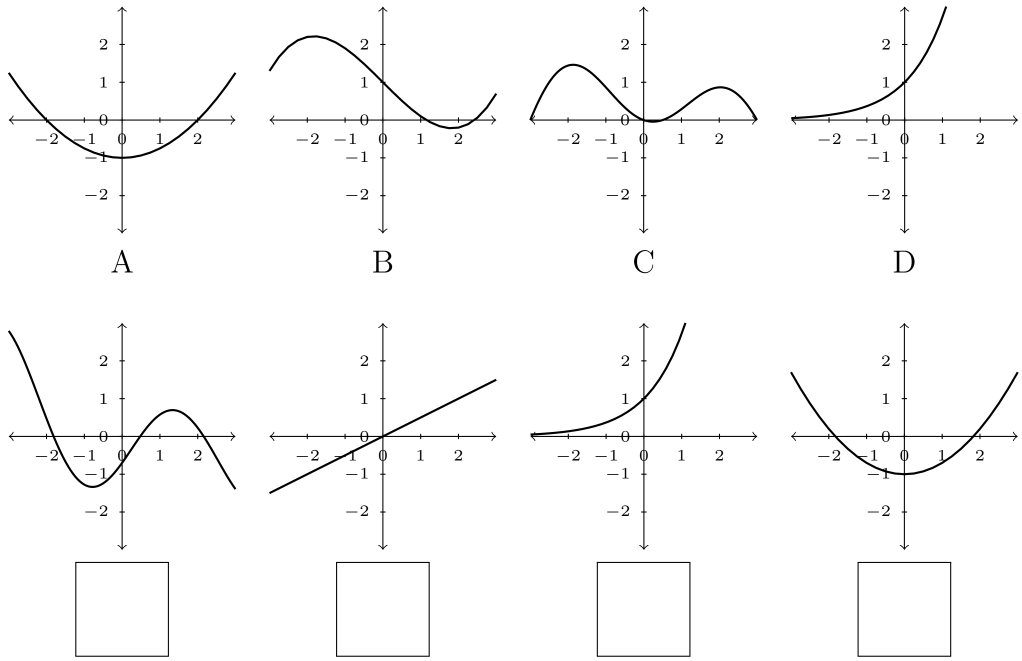
$$f(x) = \begin{cases} ax & \text{if } x < 0, \\ x^2 + 3x - 2 & \text{if } x \geq 0. \end{cases}$$

Solution: There is an error in the question as stated. One needs to replace ax by $ax - 2$. For this reason, everyone was given full credit on the question.

With the correction above made, this is how you would approach the problem. From the left, the slope of the tangent line at 0 is a. From the right, the slope of the tangent line is $2x + 3$ evaluated $x = 0$, which yields 3. Therefore we want $a = 3$.

3. (12 points) Match the functions graphed in the first row with their derivatives graphed in the second row.

 Points earned: ______ out of 32.



Solution: C A D B

4. (10 points) Find the derivative of $f(x) = \sec(x)$ using the definition of $\sec(x)$ in terms of $\cos(x)$. Simplify to show that $f'(x) = \sec(x)\tan(x)$.

Solution: Note that $\sec(x) = \frac{1}{\cos(x)}$. We compute the derivative using the chain rule (and power rule).

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\cos(x)^{-1}\right) \\ &= (-1)\cos(x)^{-2}(-\sin(x)) \\ &= \frac{\sin(x)}{\cos^2(x)} \\ &= \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} \\ &= \sec(x)\tan(x). \end{aligned}$$

5. Suppose the height of an object at t seconds is $s(t) = -t^2 + 2t + 8$ ft.
   (a) (3 points) What is the object's velocity? Give the units in which it is measured.

Solution: The velocity in ft/sec is

$$v(t) = -2t + 2.$$

(b) (3 points) What is the object's acceleration? Give the units in which it is measured.

Solution: The acceleration in ft/sec^2 is

$$a(t) = -2.$$

(c) (3 points) At what time does the object reach it's maximum height? Be sure to include the units.

Solution: When the object is at maximum height, its velocity is 0 ft/sec. We solve $-2t + 2 = 0$, and see that the object is at maximum height at $t = 1$ sec.

(d) (3 points) What is the object's maximum height? Be sure to include the units.

Solution: To get the maximum height, we compute $s(1) = -1^2 + 2(1) + 8 = 9$ ft.

6. (11 points) Suppose $y^2 - y = x^3 - x$. Find $\frac{dy}{dx}$ when $(x, y) = (2, 3)$.

Solution: Use implicit differentiation to get

$$2y\frac{dy}{dx} - \frac{dy}{dx} = 3x^2 - 1.$$

Plug in $x = 2, y = 3$ to get

$$\begin{aligned} 2(3)\frac{dy}{dx} - \frac{dy}{dx} &= 3(2^2) - 1 \\ 6\frac{dy}{dx} - \frac{dy}{dx} &= 12 - 1 \\ 5\frac{dy}{dx} &= 11 \\ \frac{dy}{dx} &= \frac{11}{5}. \end{aligned}$$

Points earned: ______ out of 25.

7. Suppose f and g are differentiable functions whose values are given below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	$\sqrt{5}$	π
2	1	3	$\sqrt{7}$	e
3	2	1	$\sqrt{11}$	$\ln(7)$

(a) (5 points) If $h(x) = 7f(x) + 5g(x)$, what is $h'(2)$?

Solution: We compute using the sum and scalar multiple rule

$$h'(x) = 7f'(x) + 5g'(x)$$
$$h'(2) = 7f'(2) + 5g'(2)$$
$$h'(2) = 7\sqrt{7} + 5e.$$

(b) (5 points) If $k(x) = \dfrac{f(x)}{g(x)}$, what is $k'(2)$?

Solution: We compute using the quotient rule

$$k(x) = \frac{f(x)}{g(x)}$$
$$k'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$
$$k'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2}$$
$$k'(2) = \frac{3\sqrt{7} - 1 \cdot e}{3^2}$$
$$k'(2) = \frac{3\sqrt{7} - e}{9}$$

(c) (5 points) If $r(x) = f(g(x))$, what is $r'(2)$?

Solution: We compute using the chain rule

$$\begin{aligned} r(x) &= f(g(x)) \\ r(x) &= f'(g(x))g'(x) \\ r(2) &= f'(g(2))g'(2) \\ r(2) &= f'(3)g'(2) \\ r(2) &= \sqrt{11}e \end{aligned}$$

8. Find the derivatives of the following functions. Use the differentiation rules that apply. You do not have to further simplify the resulting derivative. [This problem continues on the next page.]

(a) (4 points) $f(x) = (2x - 7)^9$

Solution: Use the chain rule and power rule to get

$$f'(x) = 9(2x - 7)^8(2) = 18(2x - 7)^8.$$

(b) (4 points) $s(\theta) = \sin(2\theta)$

Solution: Use chain rule to get

$$s'(\theta) = 2\cos(2\theta).$$

(c) (4 points) $h(t) = t^2 e^{\sin(t)}$

Solution: Use product rule and chain rule to get

$$h'(t) = t^2 e^{\sin(t)} \cos(t) + 2te^{\sin(t)}.$$

(d) (4 points) $g(x) = \dfrac{1 + \sin(x)}{\cos(x)}$

Solution: We give 2 possible solutions below.
First, use quotient rule to get

 Points earned: ______ out of 16.

$$\begin{aligned} g'(x) &= \frac{\cos(x)\cos(x) - (1+\sin(x))(-\sin(x))}{\cos^2(x)} \\ &= \frac{\cos^2(x) + \sin(x) + \sin^2(x)}{\cos^2(x)} \\ &= \frac{1+\sin(x)}{\cos^2(x)}. \end{aligned}$$

Method 2 involves rewriting $g(x)$ as

$$g(x) = \sec(x) + \tan(x).$$

Then

$$g'(x) = \sec(x)\tan(x) + \sec^2(x).$$

(e) (4 points) $y(t) = \sqrt{t} + \frac{1}{2t} + \frac{1}{t^3} + \sqrt{3} + \pi^e$

Solution: First rewrite $y(t)$ as

$$y(t) = t^{1/2} + \frac{1}{2}t^{-1} + t^{-3} + \sqrt{3} + \pi^e.$$

Then using power rule (and remembering that constants have 0 derivative) we compute

$$y'(t) = \frac{1}{2}t^{-1/2} - \frac{1}{2}t^{-2} - 3t^{-4}.$$