MAT 191 Test 3 November 20, 2012

Name: ____________________ Academic Integrity Signature: ____________________

I have abided by the UNCG Academic Integrity Policy.

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the 8 questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- Correct numerical answers with insufficient justification may receive little or no credit.
- Clearly distinguish your final answer from your scratch work with a box or circle.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

Page:	1	2	4	5	6	7	8	Total
Points:	10	22	22	15	8	12	11	100
Score:								

1. Evaluate the following.

   (a) (2 points) $\frac{d}{dx}(\ln(\sin(x)))$

   Solution: Don't forget the chain rule! $\frac{1}{\sin(x)} \cdot \cos(x) = \cot(x)$

   (b) (2 points) $\frac{d}{dx}(\log_7(x))$

   Solution: $\frac{1}{\ln(7)} \cdot \frac{1}{x} = \frac{1}{\ln(7)x}$

   (c) (2 points) $\frac{d}{dx}(\tan^{-1}(x^2+1))$

   Solution: Don't forget the chain rule! $\frac{1}{1+(x^2+1)^2} \cdot (2x) = \frac{2x}{1+(x^2+1)^2}$

   (d) (2 points) $\frac{d}{dx}(3^x)$

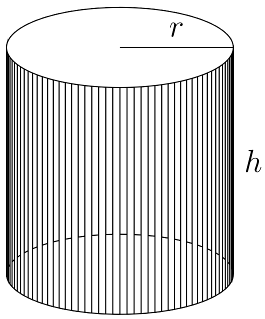
   Solution: $\ln(3)3^x$

   (e) (2 points) $\frac{d}{dx}\left(\cos^{-1}(\frac{1}{x})\right)$

   Solution: Don't forget the chain rule! $\frac{-1}{\sqrt{1-x^{-2}}} \cdot (-1)x^{-2} = \frac{1}{x^2\sqrt{1-x^{-2}}}$

 Points earned: ______ out of 10.

2. (12 points) A solid cylinder is being heated and is growing slightly. Currently, its radius is $r = 5$ cm and its height is $h = 10$ cm. If, at this time, its radius is growing at the rate of 0.2 cm/ min and its height is growing at the rate 0.1 cm/ min, then at what rate is its volume increasing? Make sure you give your answer with the right units.



Solution:

Recall the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height. We are given that $\frac{dh}{dt} = 0.1$ and $\frac{dr}{dt} = 0.2$. We are asked to compute $\frac{dV}{dt}$ when $r = 5$ cm and $h = 10$ cm.

Differentiate V with respect to time to get

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}.$$

Then plug in to get $22.5\pi \text{cm}^3/\text{min}$.

3. Evaluate the following limits:

(a) (5 points) $\lim_{x \to 0} \frac{1 - \cos(x)}{x^2}$

Solution: We compute using L'ôpital's Rule. Remember to always make sure that you have an indeterminate form.

$$\begin{aligned} \lim_{x \to 0} \frac{1 - \cos(x)}{x^2} &= \lim_{x \to 0} \frac{\sin(x)}{2x} \\ &= \lim_{x \to 0} \frac{\cos(x)}{2} \\ &= \frac{1}{2}. \end{aligned}$$

(b) (5 points) $\lim_{x \to \infty} x^{-1} \ln(x)$

Solution:

$$\begin{aligned}\lim_{x\to\infty} x^{-1}\ln(x) &= \lim_{x\to\infty}\frac{\ln(x)}{x}\\ &= \lim_{x\to\infty}\frac{1/x}{1}\\ &= 0.\end{aligned}$$

4. Let $f(x) = \sqrt{4+x}$.

   (a) (5 points) Find the linearization L of f at $a = 0$.

   Solution: We compute $f(0) = \sqrt{4} = 2$ and $f'(x) = \frac{1}{2}(4+x)^{-1/2}$ so $f'(4) = \frac{1}{4}$. We combine to get the linearization is

   $$L(x) = 2 + \frac{1}{4}x.$$

   (b) (5 points) Use L to give an approximation to $\sqrt{4.1}$.

   Solution:
   $$\sqrt{4.1} = f(0.1) \approx L(0.1) = 2 + \frac{1}{4}(0.1) = 2.025.$$

5. (12 points) Find the absolute minimum and absolute maximum of $f(x) = x^{2/3}$ on the interval $[-8, 27]$. Justify your answer. Make sure you specify where the absolute maximum and absolute minimum occur.

   Solution: Note that f is a continuous function on a closed interval. Thus by Extreme Value Theorem, f attains in global maximum and global minimum. The derivative $f'(x) = \frac{2}{3}x^{-1/3}$ is undefined at $x = 0$ and never equal to 0. We plug in the endpoints and critical points $f(-8) = 4$, $f(27) = 9$, and $f(0) = 0$. It follows that the global minimum is 0, which occurs at $x = 0$. The global maximum is 9, which occurs at $x = 27$.

Points earned: ______ out of 22.

6. Let $f(x) = x^2 + 4x + 1$.

   (a) (5 points) Does the Mean Value Theorem apply to f on the interval $[0, 2]$? Explain why or why not.

   Solution: Yes, MVT applies. Since f is a polynomial, it is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

   (b) (5 points) What is the average rate of change of f on $[0, 2]$?

   Solution: We compute $f(2) = 13$ and $f(0) = 1$. Thus

   $$\frac{\Delta y}{\Delta x} = \frac{13 - 1}{2 - 0} = 6.$$

   (c) (5 points) Find c in $(0, 2)$ so that $f'(c)$ equals the average rate of change you found in part (b).

   Solution: We compute $f'(x) = 2x + 4$. Thus if $2c + 4 = 6$, we have $c = 1$.

 Points earned: ______ out of 15.

7. Let $f(x) = 12x - x^3$ on $(-3, \infty)$.

(a) (4 points) Find the coordinates of any critical points of f.

Solution: We compute $f'(x) = 12 - 3x^2 = 3(2+x)(2-x)$. This is never undefined and equal to zero when $x = \pm 2$. We compute $f(2) = 16$ and $f(-2) = -16$. Therefore the critical points are $(2, 16)$ and $(-2, -16)$.

(b) (2 points) Find the intervals where f is increasing and those where f is decreasing.

Solution: From (a), we have $f'(x) = 3(2+x)(2-x)$. Making a sign chart with dividing lines at $x = \pm 2$, we see that $f'(x) > 0$ when $-2 < x < 2$ and $f'(x) < 0$ on $(-3, -2) \cup (2, \infty)$. It follows that f is increasing on $-2 < x < 2$ and decreasing on $(-3, -2) \cup (2, \infty)$.

(c) (2 points) Find the intervals where the graph $y = f(x)$ is concave up and those where it is concave down.

Solution: We compute $f''(x) = -6x$, which is zero at $x = 0$ and never undefined. Making a sign chart, we have that $f''(x) > 0$ for $x < 0$ and $f''(x) < 0$ for $x > 0$. It follows that the graph $y = f(x)$ is concave up on $(-3, 0)$ and concave down on $(0, \infty)$.

Points earned: ______ out of 8.

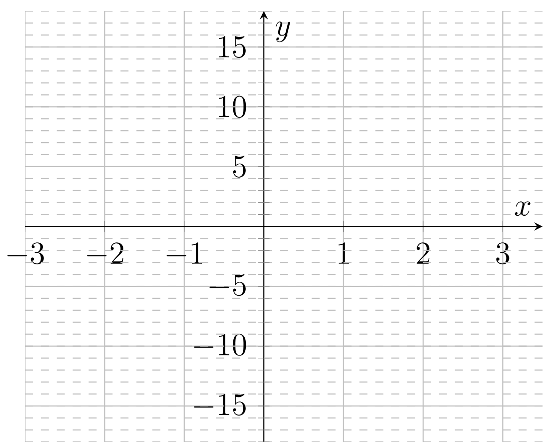
(d) (4 points) Find the inflection points of f.

Solution: From (c), we see that the graph changes concavity at $x = 0$. We compute $f(0) = 0$. It follows that the only inflection point is $(0, 0)$.

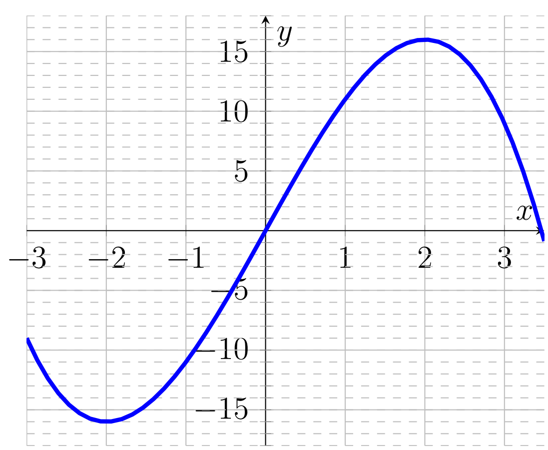
(e) (4 points) Identify all of the local extrema and where they occur. Clearly mark each as a local maximum or local minimum.

Solution: From (a), we have that the only critical points are $(2, 16)$ and $(-2, -16)$. We compute $f''(2) = -12 < 0$ and $f''(-2) = 12$. By Second Derivative Test, we get that f has a local maximum of 16 at $x = 2$ and a local minimum of -16 at $x = -2$.

(f) (4 points) Sketch the graph of $y = f(x)$ using the information from (a)-(d).

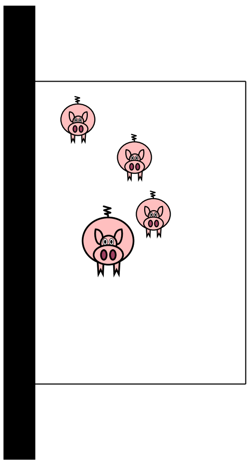


Solution:

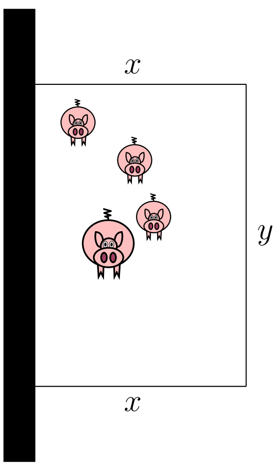


 Points earned: ______ out of 12.

8. (11 points) A farm building has a straight wall 160 feet long. Farmer Brown wants to use 160 feet of fencing to create a rectangular fenced-in pen against the wall for Daisy Mae and her three piglets—Petunia, Porkchop, and Slim. The pen will use part of the wall as one of its sides but the other three sides of the pen will have to be created using the fencing as shown below. What are the dimensions of the pen that will give the largest area? [Drawing is not to scale.]



Solution: We label some edges.



Since we have 160 feet of fencing, $2x + y = 160$. It follows that $y = 160 - 2x$. We want to maximize area

$$A = xy = x(160 - 2x) = 160x - 2x^2,$$

where $0 \le x \le 80$. Compute $\dfrac{dA}{dx} = 160 - 4x$, so the only critical point has $x = 40$. We compute $A(0) = A(80) = 0$ and $A(40) = 3200$ ft^2. It follows that the largest pen is when $x = 40$ ft and $y = 160 - 2 \cdot 40 = 80$ ft.

 Points earned: ______ out of 11.