

Name: \_\_\_\_\_ Academic Integrity Signature: \_\_\_\_\_

*I have abided by the UNCG Academic Integrity Policy.*

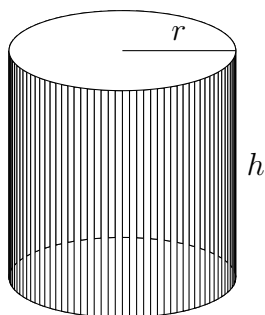
**Note:** Correct numerical answers without justification will receive little or no credit.

1. (6 points) Suppose  $f: \mathbb{R} \rightarrow \mathbb{R}$  is a differentiable, invertible function. Let  $g(x) = f^{-1}(x)$  denote the inverse of  $f$ . Fill in the table below with the correct values. Write **N** if not enough information is given to compute the value.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	5	$\frac{1}{3}$	N	N
1	4	$\frac{1}{9}$	3	$\frac{1}{15}$
2	3	10	4	$-\frac{1}{2}$
3	1	15	2	$\frac{1}{10}$
4	2	-2	1	9

**Solution:** Note that since  $f$  and  $g$  are inverse to each other, if  $f(a) = b$ , then  $g(b) = a$  and  $g'(b) = \frac{1}{f'(a)}$ . Use this to fill out the table.

2. (4 points) A solid cylinder is being heated and is growing slightly. Find an equation relating the rate of change of the volume to the rate of change of radius and rate of change of height for this metal bar. (Hint: The volume of a cylinder of radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)



**Solution:** Differentiate  $V$  using the product rule and chain rule with respect to time to get

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}.$$