MAT 191 Quiz 12 November 15, 2013

Name: ______________________ Academic Integrity Signature: ______________________

*I have abided by the UNCG Academic Integrity Policy.*

**Note**: Correct numerical answers without justification will receive little or no credit.

1. (3 points) Let $f$ be a differentiable function. Newton's method produces a sequence $x_1, x_2, x_3, \ldots$ of approximate solutions to $f(x) = 0$ given an initial guess $x_0$. Complete the formula for computing this sequence.

$$\boxed{x_{n+1}} = \boxed{\quad x_n - \frac{f(x_n)}{f'(x_n)} \quad}$$

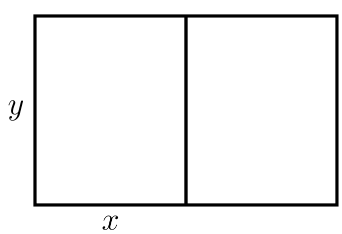
2. (3 points) The graph of $f(x) = x^3 - x + 1$ is shown below. Estimate the real root of $f$ using one iteration of Newton's method with initial guess $x_0 = -1$. i.e. Compute $x_1$.

**Solution:** We compute $f'(x) = 3x^2 - 1$, and so $f'(x_0) = f'(-1) = 2$. Furthermore, $f(x_0) = f(-1) = 1$. Then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{1}{2} = -\frac{3}{2}.$$

3. A 216 $\text{m}^2$ rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the two sides.

   (a) (2 points) Define some variables, and label them on the picture below. Find a formula for the length $F$ of fencing required.




$$F = 3y + 2x$$

   (b) (2 points) Rewrite the formula for $F$ so that it is a function of one variable. Use the constraints to find the domain of $F$.

**Solution:** The constraint is that the area is 216 $\text{m}^2$. It follows that $xy = 216$. First, we can solve for $y$ to get $y = \frac{216}{x}$. This will help write $F$ as a function of $x$

$$F = 3(\frac{216}{x}) + 2x = \frac{648}{x} + 2x.$$

Second, we can "see" the domain. Since $xy = 216$ and $x$ represents the length of one of the sides, we have the domain $0 < x < \infty$.