

Name: _____ Academic Integrity Signature: _____

I have abided by the UNCG Academic Integrity Policy. **Note:** Correct numerical answers without justification will receive little or no credit.

1. (2 points) (Derivative of logarithm)

$$\frac{d}{dx} (\log_{109} |x|) = \boxed{\frac{1}{\ln(191)} \cdot \frac{1}{x}}.$$

2. (2 points) (Derivative of exponential)

$$\frac{d}{dx} (191^x) = \boxed{\ln(191) \cdot 191^x}.$$

3. Consider the curve $x^2 + xy - y^2 = 1$.

- (a) (2 points) Verify that the point $(2, 3)$ is on the curve.

Solution: We plug $x = 2$ and $y = 3$ in to the equation of the curve and verify that we get true.

$$2^2 + 2 \cdot 3 - 3^2 = 4 - 6 + 9 = 1 \quad \checkmark$$

- (b) (4 points) Find the equation of the line that is tangent to the curve at $(2, 3)$.

Solution: To find a tangent line, we need a slope m and a point on the line. We are given the point $(2, 3)$. To find the slope, we need to compute $\frac{dy}{dx}$. Then

the slope is $m = \left. \frac{dy}{dx} \right|_{(2,3)}$.

We compute using implicit differentiation

$$\begin{aligned} x^2 + xy - y^2 &= 1 \\ 2x + x \frac{dy}{dx} + y - 2y \frac{dy}{dx} &= 0 && \text{differentiate both sides} \\ 4 + 2m + 3 - 6m &= 0 && \text{evaluate at } (2, 3) \\ -4m &= -7 \\ m &= \frac{7}{4}. \end{aligned}$$

It follows that the tangent line is

$$y - 3 = \frac{7}{4}(x - 2).$$