MAT 191 Test 2 September 30, 2013 (9:00–9:50 AM)

Name: ____________________ Academic Integrity Signature: ____________________

I have abided by the UNCG Academic Integrity Policy.

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the 9 questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- Correct numerical answers with insufficient justification may receive little or no credit.
- Clearly distinguish your final answer from your scratch work with a box or circle.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

Page:	1	2	3	4	5	6	7	Total
Points:	20	25	9	26	5	9	6	100
Score:								

1. (a) (5 points) If $f(x)$ is a function, give the definition (as a limit) of the *derivative of* $f(x)$, denoted $f'(x)$.

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) (5 points) Let $f(x) = x^2 + 3x - 2$. Use the definition to prove that $f'(x) = 2x + 3$.

Solution: We compute

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{((x+h)^2 + 3(x+h) - 2) - (x^2 + 3x - 2)}{h} \\ &= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{2} - \cancel{x^2} - \cancel{3x} + \cancel{2}}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h} \\ &= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}} \\ &= \lim_{h \to 0} 2x + h + 3 \\ &= 2x + 3. \end{aligned}$$

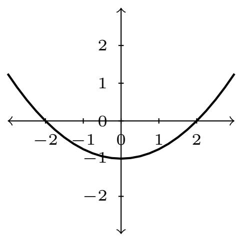
2. (10 points) Is there a value of a that will make

$$f(x) = \begin{cases} x + a & \text{if } x < 0, \\ \cos(x) & \text{if } x \geq 0 \end{cases}$$

continuous at $x = 0$? Justify.

Solution: In order for f to be continuous, we need $\lim_{x \to 0^-} f(x) = f(0)$. We have that $f(0) = \cos(0) = 1$ and $\lim_{x \to 0^-} = a$. Therefore, if we pick a to be equal to 1, f is continuous.

3. (15 points) Match the functions graphed in the first row with their derivatives graphed in the second row. No justification required.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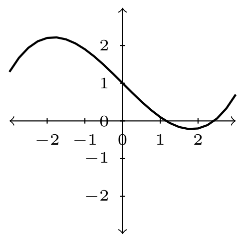
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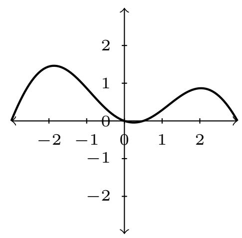
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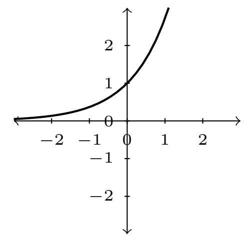
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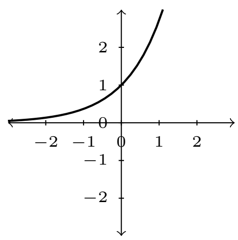
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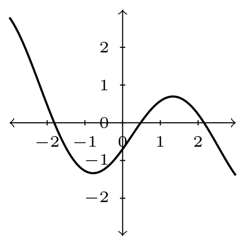
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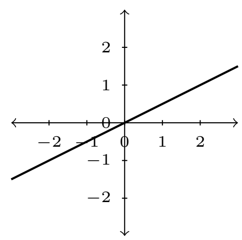
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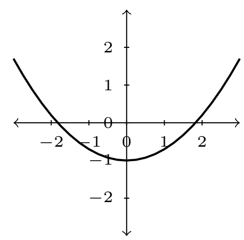
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4. (10 points) Compute the derivative of $f(x) = \tan(x)$ using the definition of $\tan(x)$ in terms of $\sin(x)$ and $\cos(x)$. Simplify to show that $f'(x) = \sec^2(x)$.

Solution:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) && \text{since } \tan(x) = \frac{\sin(x)}{\cos(x)} \\ &= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} && \text{Quotient Rule} \\ &= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} \\ &= \frac{1}{\cos^2(x)} \\ &= \sec^2(x). \end{aligned}$$

 Points earned: ______ out of 25.

5. Suppose f and g are differentiable functions whose values are given below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	$\sqrt{5}$	π
2	1	3	$\sqrt{3}$	e
3	2	1	$\sqrt{2}$	$\ln(3)$

(a) (3 points) If $h(x) = 3f(x) + 5g(x)$, what is $h'(2)$?

Solution: We compute using the sum and scalar multiple rule

$$\begin{aligned} h'(x) &= 3f'(x) + 5g'(x) \\ h'(2) &= 3f'(2) + 5g'(2) \\ h'(2) &= 3\sqrt{3} + 5e. \end{aligned}$$

(b) (3 points) If $k(x) = \dfrac{f(x)}{g(x)}$, what is $k'(2)$?

Solution: We compute using the quotient rule

$$\begin{aligned} k(x) &= \frac{f(x)}{g(x)} \\ k'(x) &= \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} \\ k'(2) &= \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} \\ k'(2) &= \frac{3\sqrt{3} - 1 \cdot e}{3^2} \\ k'(2) &= \frac{3\sqrt{3} - e}{9}. \end{aligned}$$

(c) (3 points) If $r(x) = f(g(x))$, what is $r'(2)$?

Solution: We compute using the chain rule

$$\begin{aligned} r(x) &= f(g(x)) \\ r(x) &= f'(g(x))g'(x) \\ r(2) &= f'(g(2))g'(2) \\ r(2) &= f'(3)g'(2) \\ r(2) &= \sqrt{2}e \end{aligned}$$

 Points earned: ______ out of 9.

(d) (3 points) If $p(x) = f(x)g(x)$, what is $p'(2)$?

Solution: We compute using the product rule

$$\begin{aligned} p(x) &= f(x)g(x) \\ p'(x) &= f'(x)g(x) + f(x)g'(x) \\ p'(2) &= f'(2)g(2) + f(2)g'(2) \\ p'(2) &= 3\sqrt{3} + e \end{aligned}$$

(e) (3 points) If $q(x) = x^2 g(x)$, what is $q'(2)$?

Solution: We compute using the product rule

$$\begin{aligned} q(x) &= x^2 g(x) \\ q'(x) &= 2xg(x) + x^2 g'(x) \\ q'(2) &= (2 \cdot 2)g(2) + 2^2 \cdot g'(2) \\ q'(2) &= 12 + 4e. \end{aligned}$$

6. (10 points) Let $f(x) = x^2 - 3x + 5$. Find the equation of the tangent line to $y = f(x)$ at the point $(1, 3)$.

Solution: To compute a tangent line, we need a slope and a point. The point is given as $(1, 3)$. The slope is $f'(1)$. We compute that $f'(x) = 2x - 3$, so $f'(1) = -1$. Therefore the line is

$$y - 3 = -(x - 1) \quad \text{or equivalently} \quad y = -x + 4.$$

7. (10 points) At what points does the graph of $g(x) = x^3 - 3x$ have horizontal tangents? Be sure to give the x and y coordinates of each point.

Solution: The graph has horizontal tangents where $g'(x) = 0$. We compute

$$g'(x) = 3x^2 - 3 = 3(x + 1)(x - 1).$$

It follows that the graph has horizontal tangents where $x = 1$ and $x = -1$. We plug in to g and find that the points are $(1, -2)$ and $(-1, 2)$.

 Points earned: ______ out of 26.

8. (5 points) Compute the average rate of change of $f(x) = x^3 + 1$ over the interval $[2, 3]$.

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(2)}{3 - 2} = \frac{28 - 9}{1} = 19.$$

 Points earned: ______ out of 5.

9. Find the derivatives of the following functions. Use the differentiation rules that apply. You do not have to further simplify the resulting derivative. [This problem continues on the next page.]

   (a) (3 points) $f(x) = (3x - 7)^9$

   Solution: Use the chain rule and power rule to get

   $$f'(x) = 9(3x - 7)^8(3) = 27(3x - 7)^8.$$

   (b) (3 points) $s(\theta) = \sin(2\theta - 3)$

   Solution: Use chain rule to get

   $$s'(\theta) = 2\cos(2\theta - 3).$$

   (c) (3 points) $h(t) = t^2 e^{\sin(t)}$

   Solution: Use product rule and chain rule to get

   $$h'(t) = t^2 e^{\sin(t)} \cos(t) + 2te^{\sin(t)}.$$

Points earned: ______ out of 9.

(d) (3 points) $g(x) = \dfrac{1+\sin(x)}{\cos(x)}$

Solution: We give 2 possible solutions below.
First, use quotient rule to get

$$\begin{aligned} g'(x) &= \frac{\cos(x)\cos(x) - (1+\sin(x))(-\sin(x))}{\cos^2(x)} \\ &= \frac{\cos^2(x) + \sin(x) + \sin^2(x)}{\cos^2(x)} \\ &= \frac{1+\sin(x)}{\cos^2(x)}. \end{aligned}$$

Method 2 involves rewriting $g(x)$ as

$$g(x) = \sec(x) + \tan(x).$$

Then

$$g'(x) = \sec(x)\tan(x) + \sec^2(x).$$

(e) (3 points) $y(t) = \sqrt{t} + \dfrac{1}{2t} + \dfrac{1}{t^3} + \sqrt{3} + \pi^e$

Solution: First rewrite $y(t)$ as

$$y(t) = t^{1/2} + \frac{1}{2}t^{-1} + t^{-3} + \sqrt{3} + \pi^e.$$

Then using power rule (and remembering that constants such as $\sqrt{3}$ and π^e have 0 derivative) we compute

$$y'(t) = \frac{1}{2}t^{-1/2} - \frac{1}{2}t^{-2} - 3t^{-4}.$$

 Points earned: ______ out of 6.