MAT 191 Test 3 October 28, 2013 (9:00–9:50 AM)

Name: ______________ Academic Integrity Signature: ______________

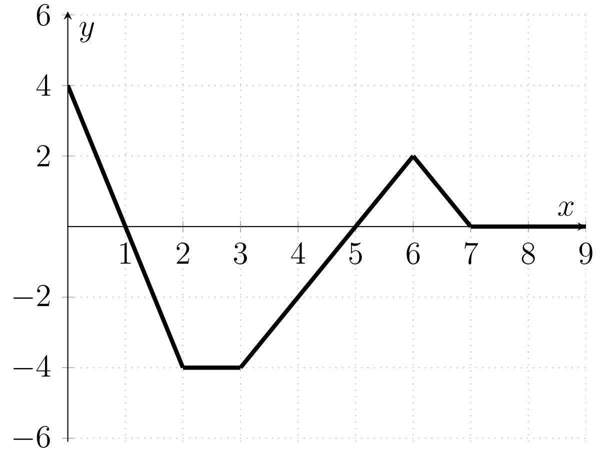
I have abided by the UNCG Academic Integrity Policy.

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the 15 questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- Correct numerical answers with insufficient justification may receive little or no credit.
- Clearly distinguish your final answer from your scratch work with a box or circle.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

Page:	1	2	3	4	5	6	7	8	Total
Points:	15	12	12	12	12	12	16	9	100
Score:									

1. The accompanying figure shows the velocity $y = v(t)$ in ft/sec of a particle moving on a line at time t seconds for $0 \leq t \leq 9$.



(a) (3 points) When is the particle moving backwards?

(b) (3 points) When is the particle speeding up?

(c) (3 points) When is the particle's acceleration positive?

2. (6 points) Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a differentiable, invertible function. Let $g(x) = f^{-1}(x)$ denote the inverse of f. Fill in the table below with the correct values. Write **N** if not enough information is given to compute the value.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	2	1	
1	0	3	0	
2	3	4		
3	$\frac{1}{2}$			$\frac{1}{4}$

3. (6 points) A rock thrown vertically upward from the surface of the moon at a velocity 24 m/sec reaches a height of $s = 24t - 0.8t^2$ meters in t seconds. How long does it take for the rock to reach its highest point?

4. (6 points) Find a formula for $\frac{dy}{dx}$ for the curve $x^3 + y^3 - 9xy = 0$.

Points earned: ______ out of 12.

5. (6 points) Let $f(x) = x\ln(x)$. Compute $f'(e)$. Simplify your answer.

6. (6 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if

$$y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}, \quad x > 1.$$

Points earned: ______ out of 12.

7. Compute the following.

   (a) (3 points) $\sec(\cos^{-1}(\frac{1}{2}))$

   (b) (3 points) $\sin^{-1}(\sin(\frac{5\pi}{6}))$

8. (6 points) Compute $\frac{dy}{dt}$, where $y = \sin^{-1}(1-t)$.

9. (6 points) Complete the statement of the *First Derivative Test for Local Extrema*. Suppose c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c, except possibly at c itself. Moving from left to right,

   1. if f' changes from negative to positive at c, then f has
      [ ] at c.
   2. if f' changes from positive to negative at c, then f has
      [ ] at c.
   3. if f' does not change sign at c, then f has
      [ ] at c.

10. (6 points) Find the absolute maximum and absolute minimum of $g(x) = xe^x$ on the interval $-2 \leq x \leq 2$. Justify your answer. Make sure you also specify where the absolute maximum and absolute minimum occur.

11. (a) (3 points) Clearly state the hypotheses of the Mean Value Theorem.

(b) (3 points) For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3 & \text{if } x = 0 \\ -x^2 + 3x + a & \text{if } 0 < x < 1 \\ mx + b & \text{if } 1 \leq x \leq 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

12. Let $f(x) = x^2 + 4x + 1$.

(a) (3 points) What is the average rate of change of f on $[0, 2]$?

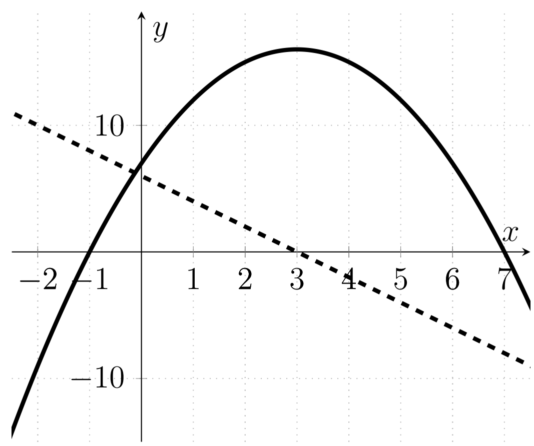
(b) (3 points) Find c in $(0, 2)$ so that $f'(c)$ equals the average rate of change you found in part (b).

 Points earned: ______ out of 12.

13. (10 points) Answer each question by circling True if it must be true and False if it is ever false. No justification is required.

- True | False: Let f be a differentiable function on $\mathbb{R}$. If the graph of f is concave up on $\mathbb{R}$, then f' is an increasing function.
- True | False: If f is a differentiable function which has a local maximum at an interior point c of its domain, then $f'(c) = 0$.
- True | False: If f is a continuous function on $\mathbb{R}$, then f attains both an absolute maximum and absolute minimum value.
- True | False: Suppose f'' is continuous on an open interval containing c. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
- True | False: Let f be an decreasing function. Then f'' is negative.

14. The graphs below show the first (solid) and second (dashed) derivative of a function $y = f(x)$.



(a) (3 points) Where is the graph of f both increasing and concave down?

(b) (3 points) Identify where the local extrema of f occur. For each, clearly identify whether it corresponds to a local maximum or local minimum.

15. Let $f(x) = x^3 - 3x + 3$.

(a) (2 points) Find the critical points of f.

(b) (2 points) Find the intervals where f is increasing.

(c) (2 points) Find all the inflection points $(c, f(c))$ of f. Find the intervals where the graph $y = f(x)$ is concave up and those where it is concave down.

(d) (3 points) Identify all of the local extrema and where they occur. Clearly mark each as a local maximum or local minimum.

Points earned: ______ out of 9.