Name: ______________________ Academic Integrity Signature: ______________________

I have abided by the UNCG Academic Integrity Policy

1. Important Conventions and Shorthand Notations

(1) Δf:
(2) ϵ:
(3) δ:
(4) $\forall$:
(5) $\exists$:
(6) $\Rightarrow$:
(7) $\rightarrow$:
(8) s.t., st, |, ":" :
(9) C^0:
(10) C^1:
(11) C^n:
(12) $f'(x)$, y', $\frac{dy}{dx}$, $\frac{df}{dx}$, $\frac{d}{dx}f(x)$, $D[f](x)$, $D_x f(x)$:

2. Functions and Intervals

(1) **Interval:**

Interval Type	**Notation**	**Meaning**
Open	(a, b)	
Semi-Open	$[a, b)$	
Semi-Open	$(a, b]$	
Closed	$[a, b]$	
All Reals	$\mathbf{R} = (-\infty, \infty)$	

(2) **Interval Combinations:**

Combination	**Notation**	**Meaning**
Union	$I \cup J$	
Intersection	$I \cap J$	
Difference	$I - J$	
Exclusions	$I - \{c_1, \dots c_n\}$	

(3) **Function f:**
   (a) Informal:
   (b) Formal:

(4) **Domain of a function f:**
   (a) Informal:
   (b) Formal:

(5) **Range (Image) of a Function f:**
   (a) Informal:
   (b) Formal:

(6) **Range (Codomain) of a Function f:**
   (a) Informal:
   (b) Formal:

(7) **Graph a Function f:**
  (a) Informal:
  (b) Formal:

(8) **Function Characteristics:**
  (a) Even:
  (b) Odd:
  (c) Periodic:
  (d) One-to-One:
  (e) Onto:
  (f) Continuous:
  (g) Differentiable:
  (h) Monotonically Increasing (Weak):
  (i) Monotonically Increasing (Strict):
  (j) Monotonically Decreasing (Weak):
  (k) Monotonically Decreasing (Strict):

(9) **Inverse function f^{-1}:**
  (a) Informal:
  (b) Formal:

(10) **Pascal's Triangle:**

(11) **General Expansion Term:**

3. Limits and Continuity

(1) **Left Side Limit:**
  (a) Informal:
  (b) Formal:

(2) **Right Side Limit:**
  (a) Informal:
  (b) Formal:

(3) **Limit:**
  (a) Informal:
  (b) Formal:

(4) **Continuity (at a point):**
  (a) Informal:
  (b) Formal:

(5) **Continuity (at a left endpoint):**
  (a) Informal:
  (b) Formal:

(6) **Continuity (at a right endpoint):**
  (a) Informal:
  (b) Formal:

(7) **Continuity (on an interval):**
  (a) Informal:
  (b) Formal:

(8) **Check List for Limits and Continuity on an Interval:**

Interval	x	**Right**	**Left**	**Full**
(a,b)	a			
	b			
	c			
$[a,b)$	a			
	b			
	c			
$(a,b]$	a			
	b			
	c			
$[a,b]$	a			
	b			
	c			

(9) **Limit Laws and Continuity Rules:**

For the table below, assume that $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$. Where continuity is asked for, assume both functions are also continuous at $x = c$.

Rule	**Expression**	$\lim_{x\to c}$	C^0 **at** c**?**	**Notes**
Sum				
Difference				
Con. Multiplier				
Product				
Quotient				
Power				
Root				

(10) **Limits of Special Functions:**

(a) **Limit of Polynomials:**

(b) **Limit of Rational Functions:**

(c) **Limit of** $(\sin\theta)/\theta$ **as** $\theta \to 0$**:**

(d) **Limit of** $(1-\cos\theta)/\theta$ **as** $\theta \to 0$**:**

(11) **Discontinuities:**
  (a) **Removable:**
  (b) **Jump:**
  (c) **Infinite:**
  (d) **Oscillating:**

(12) **Continuous Extension of a Function at a point:**
  (a) Informal:
  (b) Formal:

(13) **Comparison of Limits:**
  (a) Informal:
  (b) Formal:

(14) **The Sandwich Theorem:**
  (a) Informal:
  (b) Formal:

(15) **Intermediate Value Theorem of Continuous Functions:**
  (a) Informal:
  (b) Formal:

(16) **Limit as x Approaches ∞:**
   (a) Informal:
   (b) Formal:

(17) **Limit as x Approaches $-\infty$:**
   (a) Informal:
   (b) Formal:

(18) **Limit of ∞ at a point:**
   (a) Informal:
   (b) Formal:

(19) **Limit of $-\infty$ at a point:**
   (a) Informal:
   (b) Formal:

(20) **Asymptotes and Behavior in the Limits:**
   (a) **Vertical Asymptote:**

   (b) **Horizontal Asymptote as x Approaches ∞ or $-\infty$:**

   (c) **Oblique Asymptote as x Approaches ∞ or $-\infty$:**

   (d) **Dominant Terms:**

   (e) **Limiting Behavior as x Approaches ∞ or ∞:**

(21) **Composite Function:**
  (a) Informal:
  (b) Formal:

(22) **Limits of Composite Functions:**
  (a) Informal:
  (b) Formal:

(23) **Continuity of Composite Functions:**
  (a) Informal:
  (b) Formal:

4. Rates of Change and Derivatives

(1) **Average Rate of Change:**
   (a) Informal:
   (b) Formal:

(2) **Slope of a Curve at a point $P(x_0, f(x_0))$:**
   (a) Informal:
   (b) Formal:

(3) **Secant:**
   (a) Informal:
   (b) Formal:

(4) **Tangent:**
   (a) Informal:
   (b) Formal:

(5) **Normal:**
   (a) Informal:
   (b) Formal:

(6) **Derivative with respect to x of a function f at a point x_0:**
   (a) Informal:
   (b) Formal:

(7) **Derivative with respect to x of a function $f(x)$:**
   (a) Informal:
   (b) Formal:

(8) **Alternative Formula for the Derivative:**
   (a) Informal:
   (b) Formal:

(9) **Centered Difference Quotient:**
   (a) Informal:
   (b) Formal:

(10) **Differentiable Terminology:**
 (a) **Derivative at a Point:**
 (b) **Derivative of a Function:**
 (c) **Differentiable at** x**:**
 (d) **Differentiable:**
 (e) **Differentiable on an Open Interval:**
 (f) **Differentiable on a Closed Interval:**
 (g) **Differentiation:**
 (h) **Cusp:**
 (i) $C^0[a, b]$**:**
 (j) $C^1(a, b)$**:**
 (k) $C^n(a, b)$**:**
 (l) **Smoothness:**
 (m) **Second Order Derivative** $y'' = \frac{d^2y}{dx^2} = f''(x) = f^{(2)}(x) = D_x^2[f](x)$**:**
 (n) N**-th Order Derivative** $\frac{d^ny}{dx^n} = f^{(n)}(x) = D_x^n[f](x)$**:**

(11) **Equivalent Phrases:**
 (a) Slope of the graph of $y = f(x)$ at $x = x_0$
 (b) Slope of the tangent line at $(x_0, f(x_0))$
 (c) Derivative of $f(x)$ at $x = x_0$
 (d) (Instaneous) rate of change of $f(x)$ with respect to x at $x = x_0$
 (e) Marginal increase/change in y with respect to x (Ex. marginal cost of production)
 (f) Sensitivity of y to changes in x

(12) **Differentiablity Implies Continuity:**
 (a) Informal:
 (b) Formal:

(13) **Derivative Rules**

For the table below, assume that both the functions $f(x)$ and $g(x)$ are differentiable, as needed.

Rule	**Expression**	**Derivative**	**Notes**
Constant			
Sum			
Difference			
Constant Multiplier			
Product			
Quotient			
Power			
Reciprocal			

(14) **Important Derivatives**

Function	**Expression**	**Derivative**	**General**	**Chain Rule**
General Functions				
Power				
Exponential, Base e				
Exponential, Base a				
Logarithm, Natural				
Logarithm, Base a				
Trigonometric Functions				
Sine				
Cosine				
Tangent				
Cotangent				
Secant				
Cosecant				
Hyperbolic Functions				
Hyperbolic Sine				
Hyperbolic Consine				
Hyperbolic Tangent				
Hyperbolic Cotangent				
Hyperbolic Secant				
Hyperbolic Cosecant				
Inverse Trigonometric Functions				
Inverse Sine				
Inverse Consine				
Inverse Tangent				
Inverse Cotangent				
Inverse Secant				
Inverse Cosecant				

(15) **Physics Terminology:**

Term	**Notation**	**Meaning**	**Notes**
Position			AKA Displacement from the origin
Displacement			
Velocity			
Speed			
Acceleration			
Jerk			
Momentum			
Force			

(16) **Classic Physics Equations of Motion for Constant Acceleration:**

(a) **Position:**

(b) **Velocity:**

5. Advanced Concepts and Applications of Derivatives

(1) **Chain Rule:**
   (a) Informal:
   (b) Formal:

(2) **Implicit Differentiation:**
   (a) Informal:
   (b) Formal:

(3) **Derivative Rule for Inverse Functions:**
   (a) Informal:
   (b) Formal:

(4) **Related Rates and Change:**
In the table below, assume that $y = f(x)$, where f is sufficiently differentiable.

Concept	**Expression**
Differential:	
Derivative:	
Related Rate:	
Approximate Change:	
Actual Change:	
Relative Change:	
Linearization about x_0:	
Newton's Method for Solving $f(x) = 0$	

(5) **Global(Absolute) Maximum of a Function f:**
  (a) Informal:
  (b) Formal:

(6) **Global (Absolute) Minimum of a Function f:**
  (a) Informal:
  (b) Formal:

(7) **Local (Relative) Maximum of a Function f:**
  (a) Informal:
  (b) Formal:

(8) **Local (Relative) Minimum of a Function f:**
  (a) Informal:
  (b) Formal:

(9) **Important Locations:**
  (a) Endpoints:
  (b) Critical Points:
  (c) Inflection Points:

(10) **First Derivative Theorem:**
  (a) Informal:
  (b) Formal:

(11) **Second Derivative Test Theorem:**
  (a) Informal:
  (b) Formal:

(12) **The Extreme Value Theorem:**
(a) Informal:
(b) Formal:

(13) **Rolle's Theorem:**
(a) Informal:
(b) Formal:

(14) **The Mean Value Theorem:**
(a) Informal:
(b) Formal:

(15) **Zero Derivative Corollary:**
(a) Informal:
(b) Formal:

(16) **Difference in Antiderivatives Corollary:**
(a) Informal:
(b) Formal:

(17) **L'Hôpital's Rule:**
  (a) Informal:
  (b) Formal:

(18) **Indeterminate Powers:**
  (a) Informal:
  (b) Formal:

(19) **Cauchy's Mean Value Theorem:**
  (a) Informal:
  (b) Formal:

(20) **Functions whose derivatives have Constant Sign:**
  (a) Positive Derivative:

  (b) Negative Derivative:

(21) **Concavity:**
  (a) **Concave Up:**

  (b) **Concave Down:**

(22) **Optimization:**
  (a) **Objective Function:**

  (b) **Constraint:**

6. Graphing

(1) **Graphing Factored Polynomials - Cross-Kiss-Slide Method**
Assume that a polynomial has been factored $P(x) = A(x - a_1)^{n_1} \ldots (x - a_k)^{n_k}$.

n_i	**Graph Result**
1	
even	
odd, > 1	

(2) **Graphing Factored Rational Functions - Flip & Stick Method**
Assume that a polynomial has been factored

$$Q(x) = \frac{A(x - a_1)^{n_1} \ldots (x - a_k)^{n_k}}{(x - b_1)^{m_1} \ldots (x - b_l)^{m_l}}.$$

m_j	**Graph Result**
odd	
even	

(3) **Graphing Factored Polynomials and Rational Functions - Limiting Behaviors**
Assume that a polynomial or rational function has been factored, as above.

x **goes to**	A	$\sum_i n_i - \sum_j m_j$	**Limiting Behavior**
$+\infty$	$+$	even (positive)	
$+\infty$	$+$	odd (positive)	
$+\infty$	$+$	0	
$+\infty$	$+$	negative	
$+\infty$	-	even (positive)	
$+\infty$	-	odd (positive)	
$+\infty$	-	0	
$+\infty$	-	negative	
$-\infty$	$+$	even (positive)	
$-\infty$	$+$	odd (positive)	
$-\infty$	$+$	0	
$-\infty$	$+$	negative	
$-\infty$	-	even (positive)	
$-\infty$	-	odd (positive)	
$-\infty$	-	0	
$-\infty$	-	negative	

(4) **Graphing Rational Functions When the Numerator and Denominator have a Common Factor**

Assume that a rational function has been factored, as above, and that there is a common factor $a_p = b_q = c$.

$n_p - m_q$	**Effect on Graph**
positive	
0	
negative	

(5) **Graphing Derivatives from the Graph of Functions:**

Behavior of f	**Derivative Behavior**
Increasing	
Decreasing	
Top of a Hill	
Bottom of a Trough	
At a Cusp	

(6) **Graphing a Function Based on Its Derivatives:**

		Conclusion		
$f'(x)$	$f''(x)$	**Interior Point**	**Left Endpoint**	**Right Endpoint**
+	+			
+	0			
+	-			
0	+			
0	0			
0	-			
-	+			
-	0			
-	-			

(7) **Graphing Horizontal Asympotes (or Limiting Behaviors) using Derivatives:**

Approaching	$f''(x)$	**Conclusion**
$+\infty$	$+$	
$+\infty$	-	
$-\infty$	$+$	
$-\infty$	-	

(8) **Graphing Vertical Asymptotes using Derivatives and Limits:**

Assume $h(x) = f(x)/g(x)$, with f and g sufficiently differentiable, and $g(c) = 0$.

$f(c)$	$g(x < c)$	$g(x > c)$	**Graph Result at** $x = c$
$+$	$+$	$+$	
$+$	$+$	-	
$+$	-	$+$	
$+$	-	-	
-	$+$	$+$	
-	$+$	-	
-	-	$+$	
-	-	-	