3.4 (2, 4, 6, 8, 10, 18, 26)

(2) $s = 6t - t^2$, $0 \le t \le 6$

a) displacement $= s(6) - s(0)$
$= 0 - 0$
$= 0$ meters

average velocity $= \dfrac{0-0}{6-0} = 0$ m/s

b) speed $= \left|\dfrac{ds}{dt}\right| = |6 - 2t|$

at $t = 0$: speed $= 6$ m/sec

at $t = 6$: speed $= |6 - 12| = 6$ m/sec

acceleration $= \dfrac{dv}{dt} = -2$ m/sec^2 at both endpoints.

c) The object changes direction when $\dfrac{ds}{dt} = 6 - 2t$ changes sign. $t = 3$ sec.

(4) $s = \dfrac{t^4}{4} - t^3 + t^2$, $0 \le t \le 3$

a) displacement $= s(3) - s(0)$
$= \dfrac{3^4}{4} - 3^3 + 3^2 - 0$
$= \dfrac{81}{4} - 27 + 9$
$= 9/4$ meters

average velocity $= \dfrac{9/4}{3-0} = 3/4$ m/sec

b) speed $= \left|\dfrac{ds}{dt}\right| = |t^3 - 3t^2 + 2t|$

at $t = 0$: speed $= 0$ m/sec
at $t = 3$: speed $= 3^3 - 3\cdot 3^2 + 2\cdot 3 = 6$ m/sec

acceleration $= 3t^2 - 6t + 2$

at $t=0$: 2 m/sec^2

at $t = 3$: $3\cdot 3^2 - 6\cdot 3 + 2$
$= 27 - 18 + 2$
$= 11$ m/sec^2

c) particle changes direction when $\frac{ds}{dt} = t^3 - 3t^2 + 2t$ changes sign.

$$t^3 - 3t^2 + 2t = t(t^2 - 3t + 2) = t(t-1)(t-2)$$

	0 – 1	1 – 2	2 – 3
t	+	+	+
$t-1$	−	+	+
$t-2$	−	−	+
$f'(x)$	+	−	+
$f(x)$	/	\	/

particle changes directions at $t = 1, 2$ seconds

⑥ $s = \dfrac{25}{t+5}$, $-4 \le t \le 0$

a) displacement $= s(0) - s(-4)$

$= \dfrac{25}{5} - \dfrac{25}{-4+5}$

$= 5 - 25$

$= -20$ meters.

average velocity $= \dfrac{-20}{0-(-4)} = -5$ m/sec

b) speed $= \left|\dfrac{ds}{dt}\right| = \left|\dfrac{-25}{(t+5)^2}\right| = \dfrac{25}{(t+5)^2}$

at $t=-4$: $\dfrac{25}{(-4+5)^2} = 25$ m/sec

at $t=0$: $\dfrac{25}{(0+5)^2} = 1$ m/sec

acceleration $= \dfrac{d^2s}{dt^2} = \dfrac{50}{(t+5)^3}$

at $t=-4$: $\dfrac{50}{(-4+5)^3} = 50$ m/sec^2

at $t=0$: $\dfrac{50}{5^3} = \dfrac{2}{5}$ m/sec^2

c) since $\dfrac{ds}{dt} = \dfrac{25}{(t+5)^2} \ge 0$ the velocity never changes sign. Thus the particle never changes direction.

8 $v = t^2 - 4t + 3$

a) acceleration $a = \frac{dv}{dt} = 2t - 4$

the velocity is zero when $t^2 - 4t + 3 = (t-3)(t-1) = 0$

$t = 3, 1$

$a(3) = 2(3) - 4 = 2 \ m/sec^2$

$a(1) = 2(1) - 4 = -2 \ m/sec^2$

b) $v = t^2 - 4t + 3 = (t-3)(t-1)$

	$(0,1)$	$(1,3)$	$(3,\infty)$
$t-3$	$-$	$-$	$+$
$t-1$	$-$	$+$	$+$
v	$+$	$-$	$+$

0 1 3

particle is moving forward when $v > 0$, moving backward when $v < 0$.

forward on $(0,1) \cup (3,\infty)$ sec

backward on $(1,3)$ sec

c) the velocity is increasing when $\frac{dv}{dt} > 0$.

$\frac{dv}{dt} = 2t - 4 > 0$ when $2t > 4$

$t > 2$

velocity is increasing on $(2,\infty)$ sec.

velocity is decreasing on $(0,2)$ sec.

(10) $s = 24t - 0.8t^2$

a) $v = \frac{ds}{dt} = 24 - 1.6t$ m/sec

$a = \frac{dv}{dt} = -1.6$ m/sec^2

b) when the rock reaches its highest point, the velocity is zero.

we solve $24 - 1.6t = 0$
$24 = 1.6t$ $\Rightarrow$ $t = \frac{24}{1.6} = 15$ sec.

c) we need to compute $s(15)$

$s(15) = 24(15) - 0.8(15^2) = 180$ meters

d) half the height is 90 m.
we need to solve $s(t) = 90$

$24t - 0.8t^2 = 90$

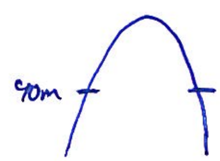
$-0.8t^2 + 24t - 90 = 0$

$t^2 - 30t - 112.5 = 0$

roots $\frac{30 \pm \sqrt{30^2 - 4(112.5)}}{2} = \frac{30 \pm \sqrt{450}}{2} = \frac{30 \pm 15\sqrt{2}}{2}$

$\frac{30 + 15\sqrt{2}}{2} \approx 25.6$ sec $\qquad \frac{30 - 15\sqrt{2}}{2} \approx 4.39$ sec.

Notice there are 2 times. This makes sense



18 a) object is moving forward when velocity is positive.

$(0,1) \cup (5,7)$ sec

moving backward when velocity is negative

$(1,5)$ sec

speeding up: velocity positive and accel. pos : $(5,6)$ sec
velocity negative and accel. neg : $(1,2)$ sec

slowing down: velocity positive and acceleration neg: $(0,1) \cup (6,7)$ sec
velocity negative and acceleration pos: $(3,5) \cup (6,7)$ sec

b) acceleration positive (slope of velocity is positive):

$(3,6)$ sec

acceleration negative: $(0,2) \cup (6,7)$ sec

acceleration zero : $(2,3) \cup (7,9)$ sec

c) greatest speed when absolute value of velocity is greatest.

$t=0$ and $t \in [2,3]$.

d) stands still on $(2,3) \cup (7,9)$

36 $Q(t) = 200(30-t)^2$.

$$\frac{dQ}{dt} = 200 \cdot 2(30-t) \cdot (-1)$$

$$= -400(30-t)$$

$$\left.\frac{dQ}{dt}\right|_{t=10} = -400(30-10) = -400(20) = -8000$$

rate of change of volume is $-8000 \frac{\text{gallons}}{\text{min}}$.

ie water draining out at $8000 \frac{\text{gallons}}{\text{min}}$.

average rate during 1st 10 min:

$Q(10) = 200(30-10)^2 = 200(20)^2 = 80000$ gall

$Q(0) = 200(30^2) = 200(900) = 180000$

$$\text{a.r.o.c.} = \frac{80000 - 180000}{10 - 0} = \frac{-100000}{10} = -10000 \frac{\text{gallons}}{\text{min}}$$

ie. $10000 \frac{\text{gallons}}{\text{min}}$ out.