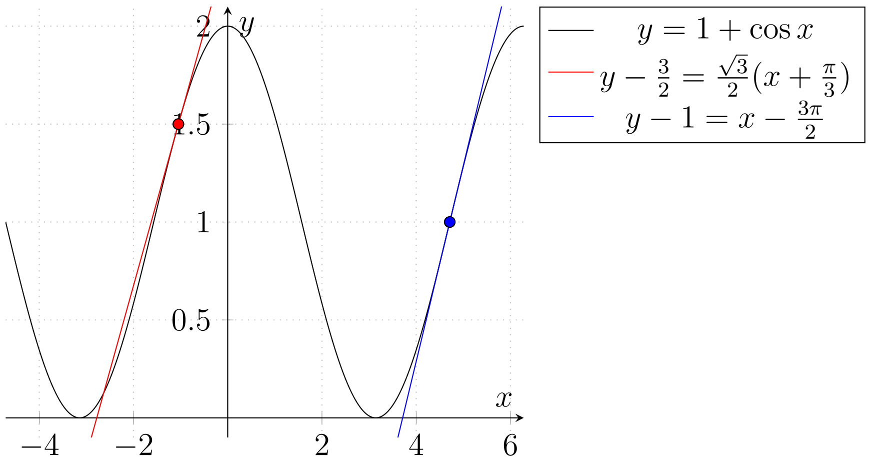
3.5 SOLUTIONS

2. $\dfrac{dy}{dx} = -3x^{-2} + 5\cos x$
4. $\dfrac{dy}{dx} = \frac{1}{2}x^{-1/2}\sec x + x^{1/2}\sec x \tan x$
6. $\dfrac{dy}{dx} = 2x\cot x - x^2\csc^2 x + 2x^{-3}$
8. $\dfrac{dy}{dx} = -\csc x \cot^2 x - \csc^3 x$
10. $\dfrac{dy}{dx} = (\cos x - \sin x)\sec x + (\sin x \cos x)\sec x \tan x$
12.

$$\begin{aligned}\frac{dy}{dx} &= \frac{(1+\sin x)(-\sin x) - \cos^2 x}{(1+\sin x)^2} \\ &= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} \\ &= \frac{-1-\sin x}{(1+\sin x)^2} \\ &= \frac{-1}{1+\sin x}\end{aligned}$$

18. $\dfrac{dy}{dx} = -\tan^2 x + (2-x)(2\tan x \sec^2 x)$
24. $\dfrac{dr}{d\theta} = \sin\theta + \theta\cos\theta - \sin\theta$
30. $\dfrac{dp}{dq} = \dfrac{(1+\tan q)\sec^2 q - \tan q \sec^2 q}{(1+\tan q)^2}$
38. We are given $y = 1 + \cos x$, so $\dfrac{dy}{dx} = -\sin x$. To find the tangent line, we need the slope and a point. To get the slope, we plug in the x-value in to $\dfrac{dy}{dx}$.
    - $x = -\frac{\pi}{3}$: The slope is $-\sin(-\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$. The point is $(-\frac{\pi}{3}, 1+\cos(-\frac{\pi}{3})) = (-\frac{\pi}{3}, \frac{3}{2})$. Therefore the tangent line is $y - \frac{3}{2} = \frac{\sqrt{3}}{2}(x + \frac{\pi}{3})$.
    - $x = \frac{3\pi}{2}$: The slope is $-\sin(\frac{3\pi}{2}) = 1$. The point is $(\frac{3\pi}{2}, 1 + \cos(\frac{3\pi}{2}) = (\frac{3\pi}{2}, 1)$. Therefore the tangent line is $y - 1 = x - \frac{3\pi}{2}$.



48. We can use direct substitution in computing limits provided there is no division by zero. Therefore

$$\begin{aligned}\lim_{x\to-\pi/6}\sqrt{1+\cos(\pi\csc x)} &= \sqrt{1+\cos(\pi\csc(-\pi/6))}\\ &= \sqrt{1+\cos(-2\pi)}\\ &= \sqrt{1+1}\\ &= \sqrt{2}.\end{aligned}$$

50. Direct substitution would yield zero in the numerator and denominator. Thus we need to simplify the expression algebraically first. Let $x = \theta - \frac{\pi}{4}$. Then as $\theta \to \frac{\pi}{4}$, we have $x \to 0$. We compute

$$\begin{aligned}\lim_{\theta\to\pi/4}\frac{\tan\theta - 1}{\theta - \frac{\pi}{4}} &= \lim_{x\to 0}\frac{\tan(x+\frac{\pi}{4}) - 1}{x}\\ &= \lim_{x\to 0}\left(\frac{\tan(x+\frac{\pi}{4}) - 1}{1-x} - 1\right)\frac{1}{x} && \text{trig identity}\\ &= \lim_{x\to 0}\left(\frac{2\tan x}{1-\tan x}\right)\frac{1}{x}\\ &= \lim_{x\to 0}\left(\frac{2\sin x}{\cos x - \sin x}\right)\frac{1}{x} && \text{mult. by } \cos x\\ &= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{2}{\cos x - \sin x}\right)\\ &= 1\cdot\frac{2}{1-0}\\ &= 2.\end{aligned}$$

54. We compute

$$\begin{aligned}\lim_{\theta\to 0}\cos\left(\frac{\pi\theta}{\sin\theta}\right) &= \cos\lim_{\theta\to 0}\frac{\pi\theta}{\sin\theta}\Big)\\ &= \cos(\pi)\\ &= -1.\end{aligned}$$

58. As we approach 0 from the left the slope of the tangent line is the slope of $x + b$, which is 1, regardless of the value of b. From the right, the slope of the tangent line is $-\sin(0) = 0$. It follows that for any value of b, we will have a corner. In particular, there is no value of b which makes g differentiable at $x = 0$.