3.6 SOLUTIONS

2. $\dfrac{dy}{dx} = 2 \cdot 3(8x-1)^2 \cdot 8 = 48(8x-1)^2$
4. $\dfrac{dy}{dx} = -\sin(-\frac{x}{3})(-\frac{1}{3}) = \frac{1}{3}\sin(-\frac{x}{3})$
6. $\dfrac{dy}{dx} = \cos(x - \cos x)(1 + \sin x)$
8. $\dfrac{dy}{dx} = -\sec(x^2+7x)\tan(x^2+7x)(2x+7)$
10. $y = u^9$, where $u = 4 - 3x$. Then
$$\frac{dy}{dx} = 9(4-3x)^8(-3) = -27(4-3x)^8$$
14. $y = \sqrt{u}$, where $u = 3x^2 - 4x + 6$. Then
$$\frac{dy}{dx} = \frac{1}{2}(3x^2-4x+6)^{-1/2}(6x-4).$$
20. $y = e^u$, where $u = \frac{2}{3}x$. Then
$$\frac{dy}{dx} = \frac{2}{3}e^{\frac{2x}{3}}.$$
22. $y = e^u$, where $u = 4\sqrt{x} + x^2$. Then
$$\frac{dy}{dx} = e^{4\sqrt{x}+x^2}(2x^{-1/2} + 2x)$$
26. $\dfrac{ds}{dt} = \frac{3\pi}{2}\cos(\frac{3\pi}{2}t) - \frac{3\pi}{2}\sin(\frac{3\pi}{2}t)$
36. Use product rule
$$\frac{dy}{dx} = 2e^{-2x} - 2(1+2x)e^{-2x}.$$
38. Use product rule
$$\frac{dy}{dx} = (18x-6)e^{x^3} + (9x^2-6x+2)(e^{x^3})(3x^2).$$
52. $\dfrac{dy}{dt} = 2\sec(\pi t)\sec(\pi t)\cos(\pi t)\pi = 2\pi\sec^2(\pi t)\cos(\pi t)$
58. $\dfrac{dy}{dt} = 3(e^{\sin(t/2)})^2 e^{\sin(t/2)}\cos(\frac{1}{2}t)\frac{1}{2} = \frac{3}{2}(e^{\sin(t/2)})^2 e^{\sin(t/2)}\cos(\frac{1}{2}t)$
62. $\dfrac{dy}{dx} = -\sin(5\sin(\frac{1}{3}t))(5\cos(\frac{1}{3}t))(\frac{1}{3}) = -\frac{5}{3}\sin(5\sin(\frac{1}{3}t))\cos(\frac{1}{3}t)$
68. We compute
$$\begin{aligned}\frac{dy}{dt} &= 4\cos^3(\sec^2(3t))(-\sin(\sec^2(3t))(2\sec(3t)\sec(3t)\tan(3t)3) \\ &= -24\cos^3(\sec^2(3t))\sin(\sec^2(3t))\sec(3t)\sec(3t)\tan(3t).\end{aligned}$$

80. Let $y = f(u)$. Then $\frac{dy}{du} = u^{-2}$. Furthermore $\frac{du}{dx} = (1-x)^{-2}$. When $x = -1$, we have $u = 1/2$. Then by the chain rule

$$\left.\frac{dy}{dx}\right|_{x=-1} = \left.\frac{dy}{du}\right|_{u=1/2} \left.\frac{du}{dx}\right|_{x=-1} = 4 \cdot \frac{1}{4} = 1.$$

82. Let $y = f(u)$. Then $\frac{dy}{du} = 1 - \frac{2}{\cos^3 u}$ and $\frac{du}{dx} = \pi$. When $x = 1/4$, $u = \pi/4$. By chain rule

$$\left.\frac{dy}{dx}\right|_{x=1/4} = \left.\frac{dy}{du}\right|_{u=\pi/4} \left.\frac{du}{dx}\right|_{x=1/4} = (1 - \frac{2}{(\sqrt{2}/2)^3})\pi = (1 - \frac{8}{\sqrt{2}})\pi.$$

88. a. We compute

$$5f'(1) - g'(1) = 5(-1/3) - (-8/3) = 1.$$

b. Use product rule

$$f'(0)g^3(0) + f(0)3g^2(0)g'(0) = 5(1^3) + 1(3(1^2)(1/3)) = 6.$$

c. Use quotient rule

$$\frac{(g(1)+1)f'(1) - f(1)g'(1)}{(g(1)+1)^2} = \frac{(-4+1)(-1/3) - (3)(-8/3)}{(-4+1)^2} = 1.$$

d. Use chain rule

$$f'(g(0))g'(0) = f'(1)(1/3) = (-1/3)(1/3) = -\frac{1}{9}.$$

e. Use chain rule

$$g'(f(0))f'(0) = g'(1)5 = (-8/3)5 = -\frac{40}{3}.$$

f. Use power rule with chain rule

$$-2(1^{11} + f(1))^{-3}(11(1^{10}) + f'(1)) = -2(4)^{-3}(32/3) = -\frac{1}{3}.$$

g. We compute

$$f'(0 + g(0))(1 + g'(0)) = f'(1)(1 + 1/3) = 5(1 + 1/3) = \frac{20}{3}.$$

92. Note that this exercise is allowing you to verify that $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$.

a. $\frac{dy}{du} = 3u^2$ and $\frac{du}{dx} = \frac{1}{2}x^{-1/2}$. Then by chain rule, we have

$$\frac{dy}{dx} = 3(\sqrt{x})^2 \frac{1}{2}x^{-1/2} = \frac{3}{2}x^{1/2}.$$

b. $\frac{dy}{du} = \frac{1}{2}u^{1/2}$ and $\frac{du}{dx} = 3x^2$. Then by the chain rule

$$\frac{dy}{dx} = \frac{1}{2}(x^3)^{-1/2}(3x^2) = \frac{3}{2}x^{-3/2}x^2 = \frac{3}{2}x^{1/2}.$$