3.7 (2, 4, 6, 10, 14, 16, 20, 30, 32, 34, 43, 50)

2 $x^3 + y^3 = 18xy$

$$3x^2 + 3y^2 \frac{dy}{dx} = 18\left(y + x\frac{dy}{dx}\right)$$

$$= 18y + 18x\frac{dy}{dx}$$

solve for $\frac{dy}{dx}$.

$$3y^2\frac{dy}{dx} - 18x\frac{dy}{dx} = 18y - 3x^2$$

$$(3y^2 - 18x)\frac{dy}{dx} = 18y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x}}$$ this simplifies to $\boxed{\frac{6y - x^2}{y^2 - 6x}}$

4 $x^3 - xy + y^3 = 1$

$$3x^2 - \left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$

$$3x^2 - y - x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$(-x + 3y^2)\frac{dy}{dx} = y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{-x + 3y^2}}$$

6 $(3xy + 7)^2 = 6y$

$$2(3xy + 7)\left(3\left(y + x\frac{dy}{dx}\right)\right) = 6\frac{dy}{dx}$$

$$(6xy + 14)\left(3y + 3x\frac{dy}{dx}\right) = 6\frac{dy}{dx}$$

$$18xy^2 + 42y + 18x^2y\frac{dy}{dx} + 42x\frac{dy}{dx} = 6\frac{dy}{dx}$$

$$(18x^2y + 42x - 6)\frac{dy}{dx} = -18xy^2 - 42y$$

$$\boxed{\frac{dy}{dx} = \frac{-18xy^2 - 42y}{18x^2y + 42x - 6}}$$

10 $xy = \cot(xy)$

$$y + x\frac{dy}{dx} = -\csc^2(xy)\left[y + x\frac{dy}{dx}\right]$$

$$y + x\frac{dy}{dx} = -\csc^2(xy)\cdot y - x\csc^2(xy)\frac{dy}{dx}$$

$$\left(x\csc^2(xy) + x\right)\frac{dy}{dx} = -y - y\csc^2(xy)$$

$$\boxed{\frac{dy}{dx} = \frac{-y - y\csc^2(xy)}{(x\csc^2(xy) + x)}}$$

14 $x\cos(2x + 3y) = y\sin x$

$$\cos(2x+3y) + x\left(-\sin(2x+3y)\left(2 + 3\frac{dy}{dx}\right)\right) = \frac{dy}{dx}\cdot\sin x + y\cos x$$

$$\cos(2x+3y) - 2x\sin(2x+3y) - 3x\sin(2x+3y)\frac{dy}{dx} = \sin x\cdot\frac{dy}{dx} + y\cos x$$

$$\left(-\sin x - 3x\sin(2x+3y)\right)\frac{dy}{dx} = y\cos x + 2x\sin(2x+3y) - \cos(2x+3y)$$

$$\boxed{\frac{dy}{dx} = \frac{y\cos x + 2x\sin(2x+3y) - \cos(2x+3y)}{-\sin x - 3x\sin(2x+3y)}}$$

16 $e^{x^2y} = 2x + 2y$

$$e^{x^2y}\left[2xy + x^2\frac{dy}{dx}\right] = 2 + 2\frac{dy}{dx}$$

$$e^{x^2y}\cdot 2xy + e^{x^2y}x^2\frac{dy}{dx} = 2 + 2\frac{dy}{dx}$$

$$\left(e^{x^2y}x^2 - 2\right)\frac{dy}{dx} = 2 - e^{x^2y}\cdot 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{(2 - 2xye^{x^2y})}{(x^2e^{x^2y} - 2)}}$$

20 $\cos r + \cot\theta = e^{r\theta}$

$-\sin r \frac{dr}{d\theta} + -\csc^2\theta = e^{r\theta}\left(\frac{dr}{d\theta}\cdot\theta + r\right)$

$-\sin r \frac{dr}{d\theta} - \csc^2\theta = \theta e^{r\theta}\frac{dr}{d\theta} + re^{r\theta}$

$(-\sin r - \theta e^{r\theta})\frac{dr}{d\theta} = \csc^2\theta + re^{r\theta}$

$$\boxed{\frac{dr}{d\theta} = \frac{\csc^2\theta + re^{r\theta}}{-\sin r - \theta e^{r\theta}}}$$

30 $(x^2+y^2)^2 = (x-y)^2$

$2(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = 2(x-y)\left(1 - \frac{dy}{dx}\right)$

at $(1,0)$: $m = \left.\frac{dy}{dx}\right|_{(1,0)}$

$2(1+0)(2+0) = 2(1-0)(1-m)$

$4 = 2(1-m)$

$2 = 1-m$

$m = -1$

at $(1,-1)$: $m = \left.\frac{dy}{dx}\right|_{(1,-1)}$

$2(1+1)(2-2m) = 2(1+1)(1-m)$

$8 - 8m = 4 - 4m$

$4 = 4m$

$\boxed{1 = m}$

32 $x^2+y^2=25$

verify $(3,-4)$ on curve:

$3^2+(-4)^2=25$ ✓

$2x+2y\frac{dy}{dx}=0$

a) $m=\left.\frac{dy}{dx}\right|_{(3,-4)}$

$2(3)+2(-4)m=0$

$6-8m=0$

$m=\frac{3}{4}$

tangent line

$$\boxed{y-(-4)=\frac{3}{4}(x-3)}$$

b) normal line is perpendicular to tangent line. Slope of normal line is $-\frac{4}{3}$.

normal line:

$$\boxed{y-(-4)=\frac{-4}{3}(x-3)}$$

34 $y^2-2x-4y-1=0$

verify $(-2,1)$ on curve:

$1^2-2(-2)-4(1)-1=1+4-4-1=0$ ✓.

$2y\frac{dy}{dx}-2-4\frac{dy}{dx}=0$

a) tangent line.

$m=\left.\frac{dy}{dx}\right|_{(-2,1)}$

$2(1)m-2-4m=0$

$-2-2m=0$

$m=-1$

tangent line

$$\boxed{y-1=-1(x-(-2))}$$

b) normal line has slope $-(-1)=1$

$$\boxed{y-1=x+2}$$

43 $y^4 = y^2 - x^2$

slope at $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$

$$4y^3 \frac{dy}{dx} = 2y \frac{dy}{dx} - 2X$$

$$m = \frac{dy}{dx}\Big|_{\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)}$$

$$4\left(\frac{\sqrt{3}}{2}\right)^3 m = \cancel{2}\frac{\sqrt{3}}{\cancel{2}} m - \cancel{2} \cdot \frac{\sqrt{3}}{\cancel{4}_2}$$

$$\frac{\cancel{12}^{3}}{\cancel{8}_2}\sqrt{3}\, m = \sqrt{3}\, m - \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} m = -\frac{\sqrt{3}}{2}$$

$$\boxed{m = -1}$$

$$m = \frac{dy}{dx}\Big|_{\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)}$$

$$4\left(\frac{1}{2}\right)^3 m = 2\left(\frac{1}{2}\right)m - \cancel{2}\left(\frac{\sqrt{3}}{\cancel{4}_2}\right)$$

$$\frac{1}{2} m = m - \frac{\sqrt{3}}{2}$$

$$-\frac{1}{2} m = -\frac{\sqrt{3}}{2}$$

$$\boxed{m = \sqrt{3}}$$

(50) $y^2 = x^3$

$2y \frac{dy}{dx} = 3x^2$

at $(1,1)$: $2m = 3$ $m = 3/2$

at $(1,-1)$: $-2m = 3$ $m = -3/2$

$2x^2 + 3y^2 = 5$

$4x + 6y \frac{dy}{dx} = 0$

at $(1,1)$: $4 + 6m = 0$
$m = -2/3$

at $(1,-1)$: $4 - 6m = 0$
$m = 2/3$

the tangent to $y^2 = x^3$ at $(1,1)$ is perpendicular to the tangent to $2x^2+3y^2=5$ at $(1,1)$.

similarly for the poin $(1,-1)$.