

3.8 (2, 4, 6, 12, 14, 20, 28, 32, 44, 50, 64, 66, 68, 70, 74, 94)

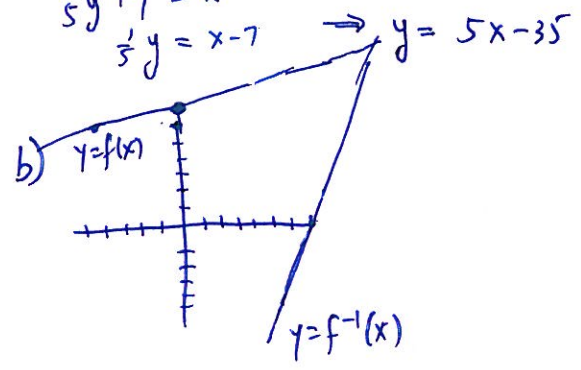
①

② $f(x) = \frac{1}{5}x + 7$, $a = -1$

$f(-1) = -\frac{1}{5} + 7 = \frac{34}{5}$

a) $y = f^{-1}(x)$
 $f(y) = x$
 $\frac{1}{5}y + 7 = x$
 $\frac{1}{5}y = x - 7$
 $y = 5x - 35$

$f^{-1}(x) = 5x - 35$



c) $\left. \frac{df}{dx} \right|_{x=-1} = \frac{1}{5}$ $\left. \frac{df^{-1}}{dx} \right|_{x=\frac{34}{5}} = 5$

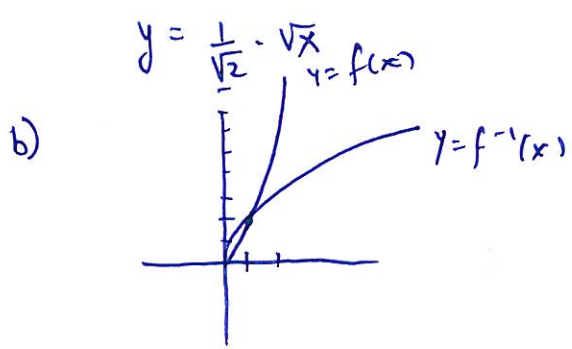
$5 = \frac{1}{\frac{1}{5}}$ ✓

④ $f(x) = 2x^2$, $x \geq 0$, $a = 5$

$f(a) = 2 \cdot 5^2 = 50$

a) $y = f^{-1}(x)$
 $f(y) = x$
 $2y^2 = x$
 $y^2 = \frac{x}{2}$

$f^{-1}(x) = \frac{1}{\sqrt{2}} \cdot \sqrt{x}$



②

$$c) f'(x) = 2(2x) = 4x$$

$$f'(5) = 4(5) = 20$$

$$\frac{df^{-1}}{dx} = \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot x^{-1/2}$$

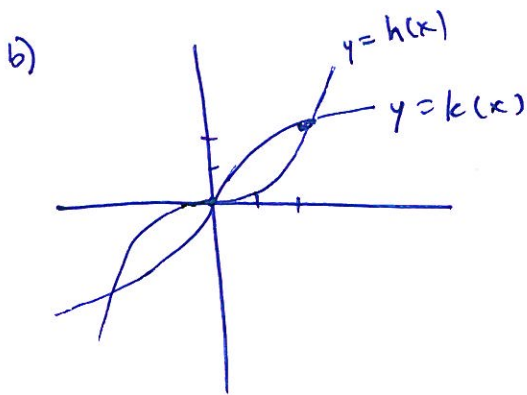
$$\frac{df^{-1}}{dx} \Big|_{x=50} = \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{50}} = \frac{1}{2} \cdot \frac{1}{\sqrt{100}} = \frac{1}{20}$$

$$\frac{1}{20} = \frac{1}{20} \quad \checkmark$$

$$⑥ a) h(x) = \frac{x^3}{4} \quad k(x) = (4x)^{1/3}$$

$$h(k(x)) = \frac{((4x)^{1/3})^3}{4} = \frac{4x}{4} = x$$

$$k(h(x)) = (4 \cdot \frac{x^3}{4})^{1/3} = (x^3)^{1/3} = x$$



$$c) h'(x) = \frac{3x^2}{4}$$

$$h'(2) = \frac{3(2^2)}{4} = \boxed{3}$$

$$h'(-2) = \boxed{3}$$

$$k'(x) = \frac{1}{3} (4x)^{-2/3} \cdot 4$$

$$k'(2) = \frac{1}{3} \cdot 8^{-2/3} \cdot 4$$

$$= \frac{4}{3} \cdot \frac{1}{4} = \boxed{\frac{1}{3}}$$

$$k'(-2) = \frac{1}{3} \cdot (-8)^{-2/3} \cdot 4$$

$$= \frac{4}{3} \cdot \frac{1}{4}$$

$$= \boxed{\frac{1}{3}}$$

(12) $y = \ln(kx) = \ln(k) + \ln x$

$$\frac{dy}{dx} = \frac{1}{x}$$

(17) $y = \ln(t^{3/2}) = \frac{3}{2} \ln(t)$

$$\frac{dy}{dt} = \frac{3}{2} \cdot \frac{1}{t}$$

(20) $y = (\ln x)^3$

$$\frac{dy}{dx} = 3(\ln x)^2 \cdot \frac{1}{x}$$

(28) $y = \frac{x \ln x}{1 + \ln x}$

$$\frac{dy}{dx} = \frac{(1 + \ln x)(\ln x + x \cdot \frac{1}{x}) - x \ln x (\frac{1}{x})}{(1 + \ln x)^2}$$

$$\frac{dy}{dx} = \frac{(1 + \ln x)^2 - \ln x}{(1 + \ln x)^2}$$

(32) $y = \ln(\sec \theta + \tan \theta)$

$$\frac{dy}{d\theta} = \frac{1}{\sec \theta + \tan \theta} \cdot [\sec \theta \tan \theta + \sec^2 \theta]$$

(34) $y = \sqrt{\frac{1}{t(t+1)}}$

$$\ln y = \frac{1}{2} (\ln(\frac{1}{t(t+1)})) = \frac{1}{2} [-\ln(t) - \ln(t+1)]$$

$$\frac{1}{y} \frac{dy}{dt} = \frac{1}{2} \left[-\frac{1}{t} - \frac{1}{t+1} \right]$$

$$\frac{dy}{dt} = \frac{1}{2} \sqrt{\frac{1}{t(t+1)}} \left[-\frac{1}{t} - \frac{1}{t+1} \right]$$

$$(50) \quad y = \frac{\theta \sin \theta}{\sqrt{\sec \theta}}$$

$$\ln y = \ln \theta + \ln(\sin \theta) - \frac{1}{2} \ln \sec \theta$$

$$\ln y = \ln \theta + \ln(\sin \theta) + \frac{1}{2} \ln(\cos \theta)$$

$$\frac{1}{y} \frac{dy}{d\theta} = \frac{1}{\theta} + \frac{1}{\sin \theta} \cdot \cos \theta + \frac{1}{2} \frac{1}{\cos \theta} \cdot (-\sin \theta)$$

$$\boxed{\frac{dy}{d\theta} = \frac{\theta \sin \theta}{\sqrt{\sec \theta}} \left[\frac{1}{\theta} + \cot \theta - \frac{1}{2} \tan \theta \right]}$$

$$(64) \quad \ln(xy) = e^{x+y}$$

$$\frac{1}{xy} \cdot \left[y + x \frac{dy}{dx} \right] = e^{x+y} \left(1 + \frac{dy}{dx} \right)$$

$$\frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = e^{x+y} + e^{x+y} \frac{dy}{dx}$$

$$\left(\frac{1}{y} - e^{x+y} \right) \frac{dy}{dx} = e^{x+y} - \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{e^{x+y} - \frac{1}{x}}{\frac{1}{y} - e^{x+y}}}$$

$$(66) \quad \tan y = e^x + \ln x$$

$$\sec^2 y \frac{dy}{dx} = e^x + \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{e^x + \frac{1}{x}}{\sec^2(y)}$$

$$\boxed{\frac{dy}{dx} = \left(e^x + \frac{1}{x} \right) \cos^2(y)}$$

(68)

$$y = 3^{-x}$$

$$\frac{dy}{dx} = \ln(3) 3^{-x} \cdot (-1)$$

(70)

$$y = 2^{s^2}$$

$$\frac{dy}{ds} = \ln(2) 2^{s^2} \cdot 2s$$

(74)

$$y = \log_3 (1 + \theta \ln(3))$$

$$\frac{dy}{d\theta} = \frac{1}{\ln(3)} \cdot \frac{1}{(1 + \ln(3) \cdot \theta)} \cdot \ln(3)$$

$$\frac{dy}{d\theta} = \frac{1}{1 + \ln(3) \cdot \theta}$$

(94)

$$y = x^{\sin x}$$

$$\ln y = \ln(x^{\sin x}) = \sin x \ln(x)$$

$$\frac{1}{y} \frac{dy}{dx} = \cos x \cdot \ln x + \frac{\sin x}{x}$$

$$\frac{dy}{dx} = x^{\sin x} \left[\cos x \cdot \ln x + \frac{\sin x}{x} \right]$$