4.1 SOLUTIONS

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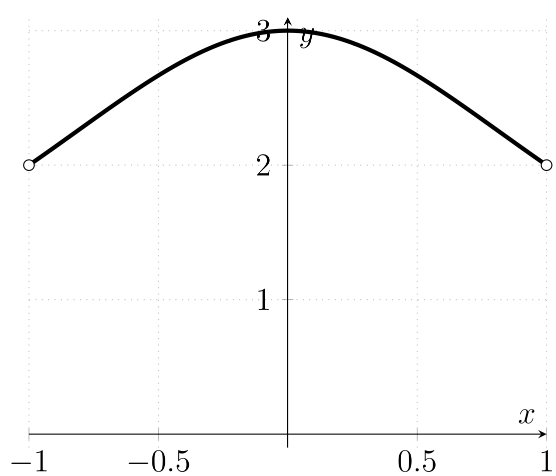
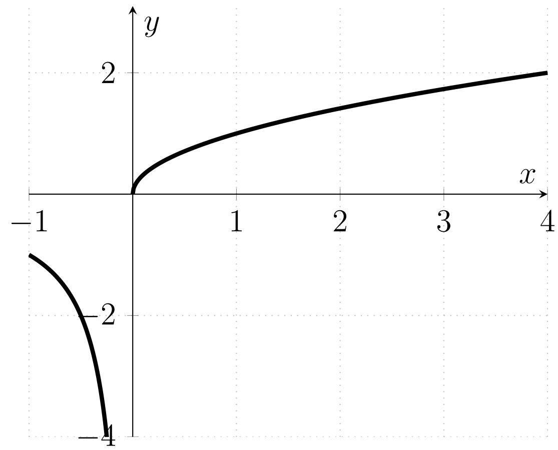
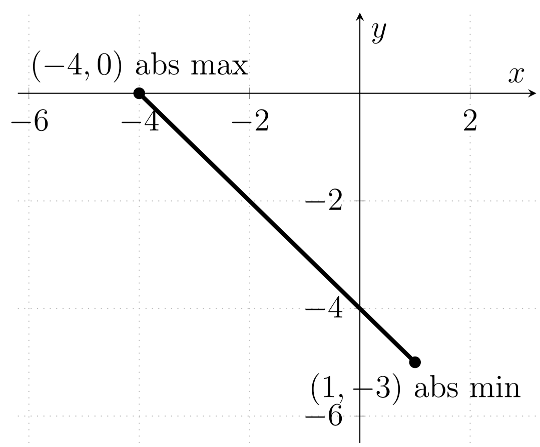
2. The function has a global maximum at c and a global minimum at b. This is consistent with the Extreme Value Theorem since the function is continuous on a closed interval and attains its global extrema.
4. The function does not have a global maximum or global minimum. Since the function is not continuous, the Extreme Value Theorem does not say anything about the function.
6. The function has a global maximum at a and a global minimum at c. Since the function is not continuous, the Extreme Value Theorem does not say anything about the function.
9. We have a global maximum of 5 at 0 and no global minimum.
12. (b) We have horizontal tangent lines at a and b, and a negative slope at c.
14. (a) We have corners at a and b and a negative slope at c.
16. See graph of $y = \frac{6}{x^2+2}$ on $(-1, 1)$ below. The function has absolute maximum of 3 at 0 and no absolute minimum. The domain is not a closed interval, so the Extreme Value Theorem does not say anything about this function.



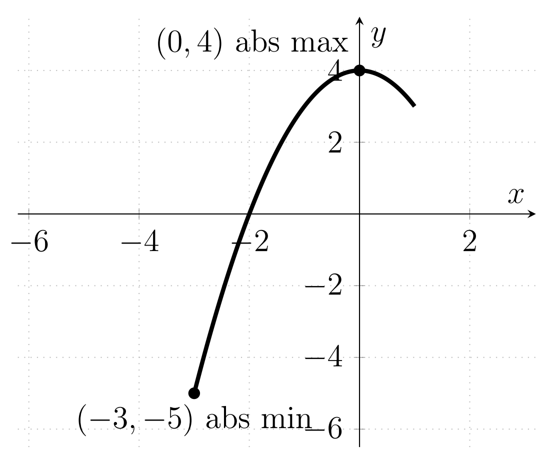
20. The function has an absolute maximum of 2 at 4 and no absolute minimum. The function is not continuous, so the Extreme Value Theorem does not say anything about this function.



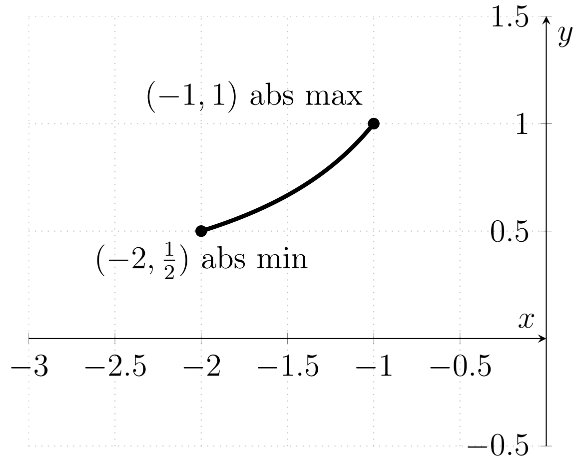
22. The global maximum is 0 and global minimum is -3.



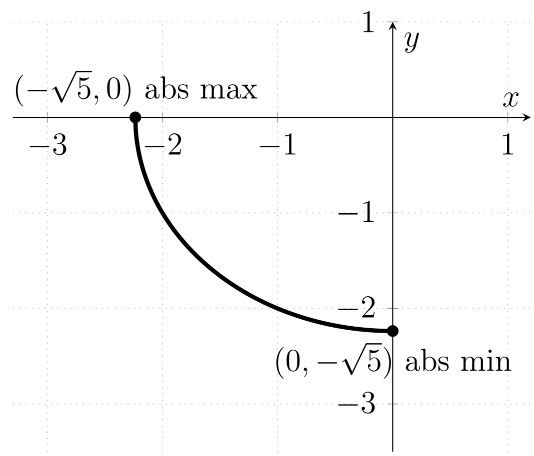
24. The global maximum is 4 and global minimum is -5.



26. The global maximum is 1 and global minimum is $\frac{1}{2}$.



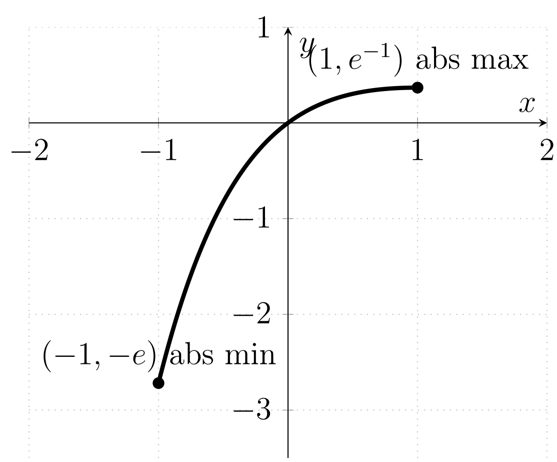
30. The global maximum is 0 and global minimum is $-\sqrt{5}$.



37. We compute

$$g'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x).$$

This is never undefined, and equal to 0 when $x = 1$. Since 1 is also an endpoint, we just evaluate g at $x = 1$ and $x = -1$ to see that the global maximum is e^{-1} and global minimum is $-e^{-1}$.



42. We want to find the absolute extrema of $f(x) = x^{5/3}$ on $[-1, 8]$. We compute
$$f'(x) = \frac{5}{3}x^{2/3}.$$
This is never undefined and equal to zero at 0. We plug in this critical point and endpoints into f and compare values
$$f(0) = 0, \quad f(-1) = -1, \quad f(8) = 2^5 = 32.$$
It follows that the absolute maximum of 32 is attained at 8 and absolute minimum of -1 is attained at -1.

44. We want to find the absolute extrema of $h(\theta) = 3\theta^{2/3}$ on $[-27, 8]$. We compute
$$h'(\theta) = 2\theta^{-1/3}.$$
This is undefined when $\theta = 0$ and never 0. We plug in this critical point and the endpoints into h and compare values
$$h(-27) = 27, \quad h(8) = 12, \quad h(0) = 0.$$
It follows that h attains an absolute maximum of 27 at -27 and absolute minimum of 0 at 0.

54. We have $y = x^3 - 2x + 4$, so
$$\frac{dy}{dx} = 3x^2 - 2 = 3(x - \sqrt{2/3})(x + \sqrt{2/3}).$$
This is never undefined and is equal to zero when $x = \pm\sqrt{2/3}$. Making a sign chart, we see that $\frac{dy}{dx} > 0$ on $(-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty)$ and $\frac{dy}{dx} < 0$ on $(-\sqrt{2/3}, \sqrt{2/3})$. Compute that
$$y(\sqrt{2/3}) = \frac{2}{3}\sqrt{\frac{2}{3}} - 2\sqrt{\frac{2}{3}} + 4 = -\frac{4}{3}\sqrt{\frac{2}{3}} + 4$$
$$y(-\sqrt{2/3}) = -\frac{2}{3}\sqrt{\frac{2}{3}} + 2\sqrt{\frac{2}{3}} + 4 = \frac{4}{3}\sqrt{\frac{2}{3}} + 4.$$
Thus we have an absolute and local maximum of $\frac{4}{3}\sqrt{\frac{2}{3}}+4$ at $-\sqrt{2/3}$ and an absolute and local minimum of $-\frac{4}{3}\sqrt{\frac{2}{3}} + 4$ at $\sqrt{2/3}$.

58. We want the local and global extrema of $y = x - 4\sqrt{x}$. Notice that the domain of the function in $[0, \infty)$. We compute $\frac{dy}{dx} = 1 - 2x^{-1/2}$. This is undefined at 0 and equal to zero when $x = 4$. We compute
$$y(0) = 0 - 0 = 0$$
$$y(4) = 4 - 4\sqrt{4} = -4.$$
Making a sign chart, we see that $\frac{dy}{dx} > 0$ on $(4, \infty)$ and $\frac{dy}{dx} < 0$ on $(0, 4)$. It follows that we have a local and absolute minimum of -4 at 4 and no absolute maximum. We have a local maximum of 0 at 0.

66. We want the local and global extrema of $y = x^2 \ln x$. Note that the domain of the function is $(0, \infty)$. We compute

$$\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x.$$

This is never undefined and is equal to zero when

$$\begin{aligned} 2x \ln x + x &= 0 \\ x(2 \ln x + 1) &= 0. \end{aligned}$$

This is zero when $x = 0$, but that is not in the domain. It is also equal to zero when $\ln x = -1/2$. This is when $x = e^{-1/2}$. We compute $y(e^{-1/2}) = -\frac{1}{2}e^{-1}$. Making a sign chart, we see that $\frac{dy}{dx} < 0$ on $(0, e^{-1/2})$ and $\frac{dy}{dx} > 0$ on $(e^{1/2}, \infty)$. It follows that we have a local and global minimum of $-\frac{1}{2e}$ at $e^{-1/2}$ and no local or global maximum.

72. Consider $y = x^{2/3}(x^2 - 4) = x^{8/3} - 4x^{2/3}$. The domain is $\mathbb{R}$. We compute

$$\frac{dy}{dx} = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3} = \frac{8}{3}\left(\frac{x^2 - 1}{x^{1/3}}\right).$$

This is undefined at 0 and is equal to zero at ± 1. We compute

$$\begin{aligned} y(0) &= 0 - 0 = 0 \\ y(1) &= 1 - 4 = -3 \\ y(-1) &= 1 - 4 = -3 \end{aligned}$$

Making a sign chart, we see that $\frac{dy}{dx} < 0$ on $(infty, -1) \cup (0, 1)$ and $\frac{dy}{dx} > 0$ on $(-1, 0) \cup (1, \infty)$. It follows that we have a local maximum of 0 at 0. We have no absolute maximum. We have local and global maximum of -3 at 1 and -1.

78. Recall that

$$|z| = \begin{cases} z & \text{if } z \geq 0, \\ -z & \text{if } z < 0. \end{cases}$$

It follows that to understand $f(x) = |x^3 - 9x|$, we first need to understand where $x^3 - 9x$ is positive and where it is negative. We factor

$$x^3 - 9x = x(x + 3)(x - 3)$$

and see that it is zero at $0, 3, -3$ and never undefined. We make a sign chart and see that $x^3 - 9x < 0$ on $(-\infty, -3) \cup (0, 3)$ and $x^3 - 9x > 0$ on $(-3, 0) \cup (3, \infty)$. It follows that

$$f(x) = \begin{cases} x^3 - 9x & \text{if } x \in (-3, 0) \cup (3, \infty), \\ -x^3 + 9x & \text{if } x \in (-\infty, -3) \cup (0, 3). \end{cases}$$

From this, we see that

$$f'(x) = \begin{cases} 3x^2 - 9 & \text{if } x \in (-3, 0) \cup (3, \infty), \\ -3x^2 + 9 & \text{if } x \in (-\infty, -3) \cup (0, 3). \end{cases}$$

The points $0, 3, -3$ require more work.

a. As we approach 0 from the left, the slope of the tangent line is -9. As we approach 0 from the right, the slope of the tangent line is 9. It follows that we have a corner there, and so $f'(0)$ does not exist,
b. As we approach 3 from the left, the slope of the tangent line is $-27+9=-18$. As we approach 3 from the right, the slope of the tangent line is $27-9=18$. As above, we have a corner and so $f'(3)$ does not exist.
c. As we approach -3 from the left, the slope of the tangent line is $-27+9=-18$. As we approach from the right, the slope of the tangent line is $27-9=18$. As above, we have a corner and so $f'(-3)$ does not exist.

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