4.3 (2, 6, 10, 14, 16, 20, 22, 36, 44, 70) ①

② $f'(x) = (x-1)(x+2)$

a) Critical points are where $f'(x) = 0$ or undefined.

$\Rightarrow$ critical points at $x = 1, -2$.

b) f is increasing when $f' > 0$, decreasing when $f' < 0$.

		-2		1	
$x-1$	$-$		$-$		$+$
$x+2$	$-$		$+$		$+$
$f'(x)$	$+$		$-$		$+$
$f(x)$	/		\		/

f is increasing on $(-\infty, -2) \cup (1, \infty)$

f is decreasing on $(-2, 1)$

c) We use the First Derivative test for local extrema.

f has a local minimum at $x = -2$ should say maximum

f has a local maximum at $x = 1$ should say minimum

⑥ $f'(x) = (x-7)(x+1)(x+5)$

a) reason as above. critical pts at $x = 7, -1, -5$.

b)

		-5		-1		7	
$x-7$	$-$		$-$		$-$		$+$
$x+1$	$-$		$-$		$+$		$+$
$x+5$	$-$		$+$		$+$		$+$
$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	\		/		\		/

f is increasing on $(-5, -1) \cup (7, \infty)$

f is decreasing on $(-\infty, -5) \cup (-1, 7)$

c) using First Derivative test for local extrema,

f has local minima at $x = -5$ and $x = 7$

f has local maximum at $x = -1$.

(10) $f'(x) = 3 - \frac{6}{\sqrt{x}}$, $x \neq 0$ [Note: $x > 0$]

a) $f'(x)$ is undefined at $x \le 0$, but this is not in the domain of f so it does not give a critical point.

$f'(x) = 0$ when $3 = \frac{6}{\sqrt{x}} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

critical pt $x = 4$.

b)

$3 - \frac{6}{\sqrt{x}}$	undef	$-$	$+$
	0	4	
$f'(x)$		$-$	$+$
$f(x)$		$\searrow$	$\nearrow$

f is decreasing on $(0, 4)$

f is increasing on $(4, \infty)$.

c) By First Derivative test, f has local minimum at $x = 4$.

(14) $f'(x) = (\sin x + \cos x)(\sin x - \cos x)$ $0 \le x \le 2\pi$

a) $f'(x)$ is never undefined

$f'(x) = 0$ when $\sin x + \cos x = 0$ or $\sin x - \cos x = 0$

$\sin x = -\cos x$

we want y coord on unit circle to equal x coord.

$x = 3\pi/4 , 7\pi/4$

$\sin x = \cos x$

we want y coord on unit circle to equal x coord

$x = \pi/4 , 5\pi/4$

Critical points $x = \pi/4 , \frac{3\pi}{4} , 5\pi/4 , 7\pi/4$

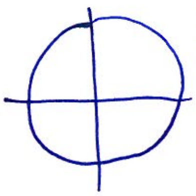
b)

$\sin x + \cos x$	+	+	−	−	+
$\sin x - \cos x$	−	+	+	−	−

$0 \quad \frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{5\pi}{4} \quad \frac{7\pi}{4} \quad 2\pi$

$f'(x)$ − + − + −

$f(x)$ ↘ ↗ ↘ ↗ ↘



f is increasing on $(\pi/4, 3\pi/4) \cup (\frac{5\pi}{4}, 7\pi/4)$

f is decreasing on $(0, \pi/4) \cup (3\pi/4, 5\pi/4) \cup (7\pi/4, 2\pi)$

c) By First Derivative test for local extreme

f has local minimum at $x = \pi/4, 5\pi/4$

f has local maximum at $x = 3\pi/4, 7\pi/4$

Additionally, we need to look at end points. Since $f'(x) < 0$ to the left of 0,

f has a local maximum at $x = 0$ f has a local minimum at $x = 2\pi$.

Since $f'(x) < 0$ to the right of 2π,

(18) a) f is increasing on $(-1.5, 1) \cup (2, 4) \cup (-4, -3)$

f is decreasing on $(-3, -1.5) \cup (1, 2)$

b) f has local minima at $(-4, 0)$, $(-1.5, -1)$, $(2, 0)$

f has absolute minimum at $(-1.5, -1)$

f has local maxima at $(-3, 1)$, $(1, 1)$, $(4, 2)$

f has absolute maximum at $(4, 2)$

(20) $g(t) = -3t^2 + 9t + 5$

a) $g'(t) = -6t + 9$

$g'(t) = 0$ when $t = 9/6 = \frac{3}{2}$

$-6t+9$ + − (at $\frac{3}{2}$)

$f(t)$ ↗ ↘

f is increasing on $(-\infty, 3/2)$

f is decreasing on $(3/2, \infty)$

b) by the first Derivative test for local extrema, g has local maximum at $(3/2, g(3/2))$

$$g(3/2) = -3(3/2)^2 + 9(3/2) + 5$$
$$= \frac{-27}{4} + \frac{27}{2} + 5$$
$$= \frac{47}{4}$$

local maximum at $(3/2, 47/4)$

global maximum at $(3/2, 47/4)$ since g is decreasing to the right of $3/2$ and increasing to the left of $3/2$.

(22) $h(x) = 2x^3 - 18x$

a) $h'(x) = 6x^2 - 18$

$h'(x) = 0$ when $6(x^2 - 3) = 0$

$x = \pm\sqrt{3}$

		$-\sqrt{3}$		$\sqrt{3}$	
6	$+$		$+$		$+$
$(x+\sqrt{3})$	$-$		$+$		$+$
$(x-\sqrt{3})$	$-$		$-$		$+$
$f'(x)$	$+$		$-$		$+$
$f(x)$	/		\		/

f is increasing on $(-\infty, \sqrt{3}) \cup (\sqrt{3}, \infty)$

f is decreasing on $(-\sqrt{3}, \sqrt{3})$

b) By First Derivative Test for local extrema local maximum at $(-\sqrt{3}, h(-\sqrt{3}))$

$h(-\sqrt{3}) = 2(-\sqrt{3})^3 - 18(-\sqrt{3}) = 12\sqrt{3}$. local max $(-\sqrt{3}, 12\sqrt{3})$

local minimum at $(\sqrt{3}, h(\sqrt{3})) = (\sqrt{3}, -12\sqrt{3})$

No absolute maximum or minimum.

36) $f(x) = \dfrac{x^3}{3x^2+1}$

a) $$f'(x) = \frac{(3x^2+1)(3x^2) - x^3(6x)}{(3x^2+1)^2}$$

$$= \frac{9x^4 + 3x^2 - 6x^4}{(3x^2+1)^2}$$

$$= \frac{3x^4 + 3x^2}{(3x^2+1)^2}$$

$f'(x)$ is undefined when $(3x^2+1)^2 = 0$. Never.

$f'(x)$ is zero when $3x^4 + 3x^2 = 0$

$$\underbrace{3x^2}_{=0 \text{ when } x=0}\underbrace{(x^2+1)}_{\text{never } 0}$$

		0	
$3x^2$	+		+
x^2+1	+		+
$(3x^2+1)^2$	+		+
$f'(x)$	+	0	+
$f(x)$	↗		↗

f is increasing on $(-\infty, 0) \cup (0, \infty)$. Perhaps better is $(-\infty, \infty)$.

f is never decreasing.

b) By First Derivative Test, f has no local extrema.

Similarly, no absolute extrema.

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$f(x) = x^2 \ln x$

The domain is $(0, \infty)$.

a) $f'(x) = 2x \ln x + x^2 \left(\frac{1}{x}\right)$

$= 2x \ln x + x$

$= x(2\ln x + 1)$

This is never undefined and equal to zero when $2\ln x + 1 = 0$

$\ln x = -\frac{1}{2} \Rightarrow x = e^{-1/2}$. [Note: $x = 0$ is not in domain.]

make a sign chart:

	$(0, e^{-1/2})$	$(e^{-1/2}, \infty)$
x	$+$	$+$
$2\ln x + 1$	$-$	$+$
$f'(x)$	$-$	$+$
$f(x)$	$\searrow$	$\nearrow$

f is increasing on $(e^{-1/2}, \infty)$

f is decreasing on $(0, e^{-1/2})$

b) we have local and absolute minimum of $f(e^{-1/2}) = e^{-1}\left(-\frac{1}{2}\right) = -\frac{1}{2e}$

at $e^{-1/2}$.

no maximum.

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a) $h(0) = 0$ $\Rightarrow$ graph goes through origin

$-2 \leq h(x) \leq 2$ $\Rightarrow$ y-coord between -2 and 2

$h'(x) \to \infty$ as $x \to 0^-$

$h'(x) \to \infty$ as $x \to 0^+$

as x approaches 0 from either side, tangent approaches vertical