4.4 (2, 6, 8, 10, 12, 14, 24, 40, 52, 58, 60, 62, 79, 82, 90, 104, 106)

② $y = \frac{x^4}{4} - 2x^2 + 4$

$\frac{dy}{dx} = x^3 - 4x = x(x^2-4) = x(x+2)(x-2)$ ⇒ critical pts 0, 2, −2.

$\frac{d^2y}{dx^2} = 3x^2 - 4 = 3\left(x^2 - \frac{4}{3}\right) = 3\left(x+\frac{2}{\sqrt{3}}\right)\left(x-\frac{2}{\sqrt{3}}\right)$

		$-\frac{2}{\sqrt{3}}$		$\frac{2}{\sqrt{3}}$	
3	+		+		+
$x+\frac{2}{\sqrt{3}}$	−		+		+
$x-\frac{2}{\sqrt{3}}$	−		−		+
$\frac{d^2y}{dx^2}$	+		−		+

Concave up $\left(-\infty, \frac{-2}{\sqrt{3}}\right) \cup \left(\frac{2}{\sqrt{3}}, \infty\right)$
Concave down $\left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

inflection points $(c, f(c))$ where $c = \frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$.

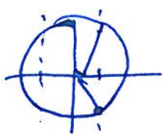
$f\left(\frac{2}{\sqrt{3}}\right) = \frac{16}{9} = f\left(-\frac{2}{\sqrt{3}}\right)$

inflection pts $\left(\frac{-2}{\sqrt{3}}, 16/9\right), \left(\frac{2}{\sqrt{3}}, 16/9\right)$

By 2nd Derivative test, we have local maximum at 0. $f(0) = 4$.
we have local minimum at 2 and −2. $f(2) = 0 = f(-2)$.

(6) $y = \tan x - 4x \quad , \quad -\pi/2 < x < \pi/2$

$\frac{dy}{dx} = \sec^2 x - 4 = (\sec x + 2)(\sec x - 2)$



$\frac{dy}{dx} = 0$ when $\sec x = \pm 2 \Rightarrow \cos x = \pm \frac{1}{2}$

critical pts $\pi/3, -\pi/3$

$\frac{d^2y}{dx^2} = 2 \sec x \sec x \tan x$

$= 2 \cdot \frac{\sin x}{\cos^3 x}$

not undefined in domain

equal 0 when $\sin x = 0$

$x = 0$

	$(-\pi/2, 0)$	$(0, \pi/2)$
2	+	+
$\sin x$	−	+
$\cos^3 x$	+	+
$f''(x)$	−	+

$\boxed{\text{Concave up on } (0, \pi/2) \quad \text{Concave down on } (-\pi/2, 0)}$

inflection pt $(0, \tan 0 - 4(0)) = \boxed{(0, 0).}$

By second derivative test for local extrema, we have local max at

$-\pi/3$ of $\tan(-\pi/3) - 4(-\pi/3) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} + 4\pi/3$

$= -\sqrt{3} + 4\pi/3.$

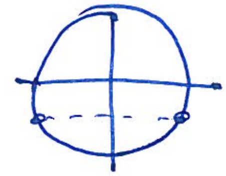
we have local min at $\pi/3$ of $\tan(\pi/3) - 4(\pi/3) = \sqrt{3} - 4\pi/3.$

⑧ $y = 2\cos x - \sqrt{2}\,x \qquad -\pi \le x \le 3\pi/2$

$\frac{dy}{dx} = -2\sin x - \sqrt{2}$

$\frac{dy}{dx} = 0$ when $-2\sin x - \sqrt{2} = 0$

$\sin x = \frac{-\sqrt{2}}{2}$

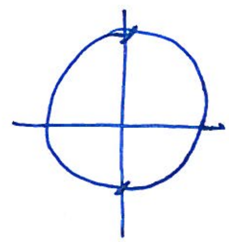


critical pts $-3\pi/4,\ -\pi/4,\ 5\pi/4$

$\frac{d^2y}{dx^2} = -2\cos x$

this is never undefined.

this is zero when $x =$



$-\pi/2,\ \pi/2,\ 3\pi/2$

	$(-\pi, -\pi/2)$	$(-\pi/2, \pi/2)$	$(\pi/2, 3\pi/2)$
-2	$-$	$-$	$-$
$\cos x$	$-$	$+$	$-$
$f''(x)$	$+$	$-$	$+$

concave up on $(-\pi, -\pi/2) \cup (\pi/2, 3\pi/2)$.
concave down on $(-\pi/2, \pi/2)$

inflection pts:

$f(-\pi/2) = 2\cos(-\pi/2) - \sqrt{2}(-\pi/2)$

$= 0 + \frac{\sqrt{2}\,\pi}{2}$

$= \frac{\sqrt{2}\,\pi}{2}$

$\boxed{\left(-\pi/2, \frac{\sqrt{2}\,\pi}{2}\right)}$

$f(\pi/2) = 2\cos(\pi/2) - \sqrt{2}(\pi/2)$

$= -\sqrt{2}\,\frac{\pi}{2}$

$\boxed{\left(\pi/2, \frac{-\sqrt{2}\,\pi}{2}\right)}$

By second derivative test for local extrema,

f has local min at $-3\pi/4$ of

$$f(-3\pi/4) = 2\cos(-3\pi/4) - \sqrt{2}\,(-3\pi/4)$$

$$= \cancel{2}\left(-\frac{\sqrt{2}}{\cancel{2}}\right) + \frac{3\sqrt{2}\,\pi}{4}$$

$$= -\sqrt{2} + \frac{3\sqrt{2}\,\pi}{4}$$

f has local min at $5\pi/4$ of

$$f(5\pi/4) = 2\cos(5\pi/4) - \sqrt{2}\,(5\pi/4)$$

$$= -\sqrt{2} - 5\sqrt{2}\,\pi/4$$

f has local max at $-\pi/4$ of

$$f(-\pi/4) = 2\cos(-\pi/4) - \sqrt{2}\,(-\pi/4)$$

$$= \cancel{2}\frac{\sqrt{2}}{\cancel{2}} + \frac{\sqrt{2}\,\pi}{4}$$

$$= \sqrt{2} + \frac{\sqrt{2}}{4}\pi \,.$$

(10) $f(x) = 6 - 2x - x^2$.

Domain is $\mathbb{R}$.

$f'(x) = -2 - 2x = -2(x+1)$

Critical pt -1

-2	$-$		$-$
$x+1$	$-$		$+$
$f'(x)$	$+$	-1	$-$
$f(x)$	$\nearrow$		$\searrow$

f is increasing on $(-\infty, -1)$

f is decreasing on $(-1, \infty)$.

f has local max at -1. $f(-1) = 6 - 2(-1) - (-1)^2$
$= 6 + 2 - 1$
$= 7$

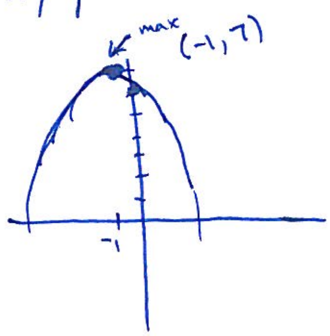
this is also absolute max since f is increasing to left and decreasing to right.

$f''(x) = -2$

f is always concave down.

no inflection points

no asymptotes.



(12) $y = f(x) = x(6-2x)^2$ domain is $\mathbb{R}$.

$f'(x) = (6-2x)^2 + x(2)(6-2x)(-2)$

$= (6-2x)(6-2x-4x)$

$= (6-2x)(6-6x)$

$= 12(3-x)(1-x)$

critical pts 3, 1.

12	$+$		$+$		$+$
$3-x$	$+$		$+$		$-$
$1-x$	$+$		$-$		$-$
$f'(x)$	$+$	1	$-$	3	$+$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$

f is increasing on $(-\infty, 1) \cup (3, \infty)$

f is decreasing on $(1, 3)$

By First Derivative test, f has local max at $x=1$

$$f(1) = 1(6-2)^2 = 16$$

f has local min at $x = 3$

$$f(3) = 3(6-6)^2 = 0$$

$$f''(x) = 12\left[-(1-x) + -(3-x)\right]$$
$$= 12\left[-1 + x - 3 + x\right]$$
$$= 12\left[2x - 4\right]$$
$$= 24(x-2)$$

never undef.

equal 0 when $x = 2$

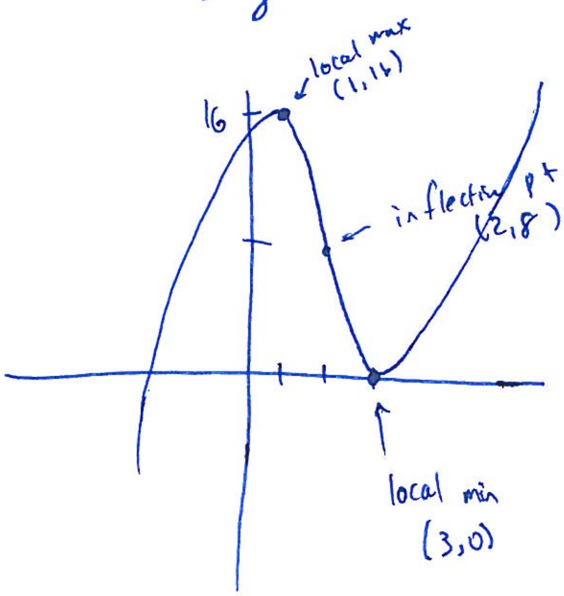
24	$+$		$+$
$x-2$	$-$		$+$
$f''(x)$	$-$	2	$+$

Concave up on $(2, \infty)$

Concave down on $(-\infty, 2)$

inflection pt $(2, 8)$

$$f(2) = 2(6-2(2))^2$$
$$= 2(2)^2$$
$$= 8$$



(14) $f(x) = 1 - 9x - 6x^2 - x^3$

$f'(x) = -9 - 12x - 3x^2$ y - intercept 1

$= -3(x^2 + 4x + 3)$

$= -3(x+3)(x+1)$

critical pts $-3, -1$

		-3		-1	
-3	$-$		$-$		$-$
$x+3$	$-$		$+$		$+$
$x+1$	$-$		$-$		$+$
$f'(x)$	$-$		$+$		$-$
$f(x)$	$\searrow$		$\nearrow$		$\searrow$

increasing on $(-3, -1)$

decreasing on $(-\infty, -3) \cup (-1, \infty)$.

f has local max at -1: $f(-1) = 1 + 9 - 6 + 1 = 5$

f has local min at -3: $f(-3) = 1 + 27 - 54 + 27 = 1$

$f''(x) = -12 - 6x = -6(x+2)$ equal 0 when $x = -2$

		-2	
-6	$-$		$-$
$x+2$	$-$		$+$
$f''(x)$	$+$		$-$

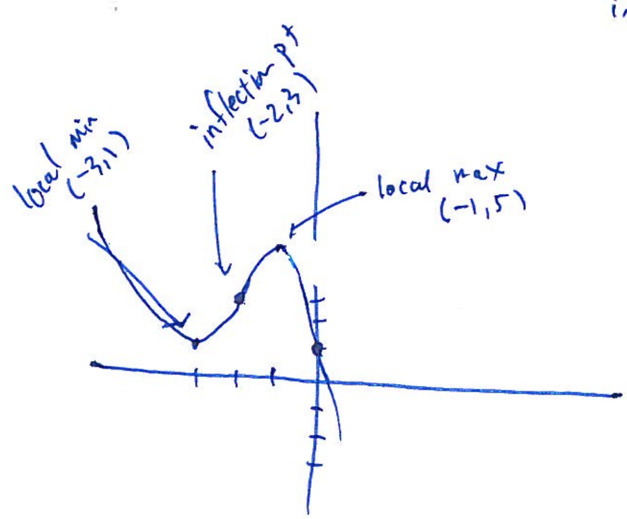
f is concave up on $(-\infty, -2)$

concave down on $(-2, \infty)$

inflection pt when $x = -2$:

$f(-2) = 1 + 18 - 24 + 8 = 3$

$(-2, 3)$

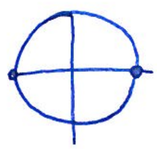


y-intercept 1

(24) $f(x): y = x - \sin x$, $0 \le x \le 2\pi$ ← domain

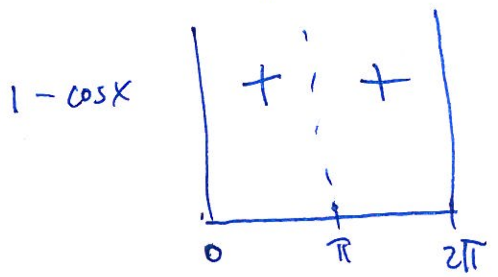
y - intercept
0

$f'(x) = 1 - \cos x$



$f'(x) = 0$ when $\cos x = 1$

$x = 0, \pi, 2\pi$ critical pts.



f is increasing on $(0, \pi) \cup (\pi, 2\pi)$.

no local extreme at π.

By First Derivative Test.

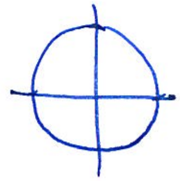
$f(0) = 0$ ← absolute min

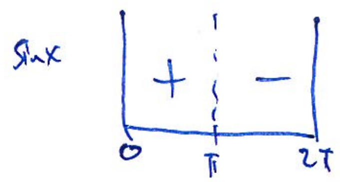
$f(\pi) = \pi$

$f(2\pi) = 2\pi$ ← absolute max

$f''(x) = \sin x$

$\sin x = 0$ at $x = 0, \pi, 2\pi$.

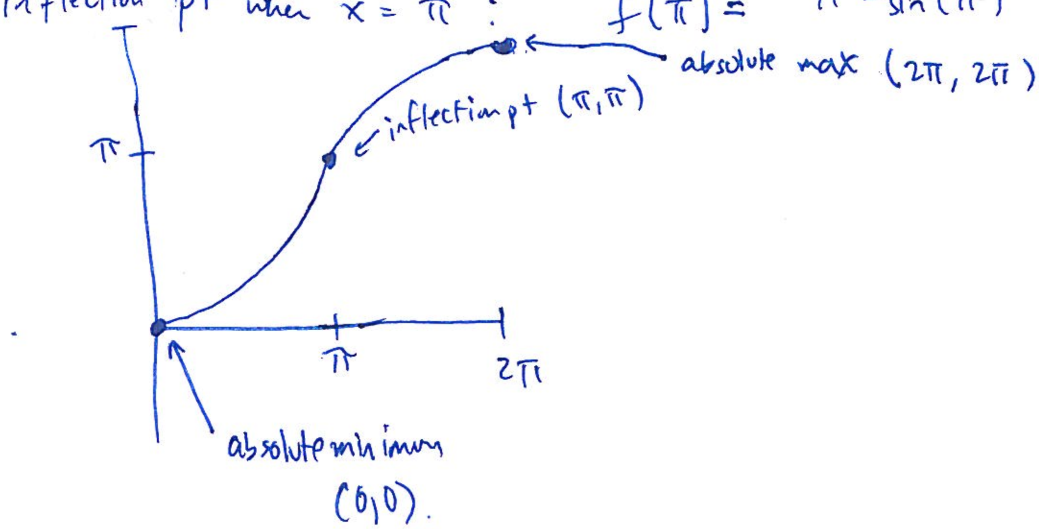




f is concave up on $(0, \pi)$

f is concave down on $(\pi, 2\pi)$.

inflection pt when $x = \pi$: $f(\pi) = \pi - \sin(\pi) = \pi$



40 $f(x) = x^2 + 2x^{-1}$ domain $(-\infty, 0) \cup (0, \infty)$

$f'(x) = 2x - 2x^{-2}$

$= 2x - \frac{2}{x^2} = \frac{2x^3 - 2}{x^2}$

undefined when $x = 0$, but this is not in the domain.

equal zero when $2x^3 - 2 = 2(x^3 - 1) = 0$

$x = 1$. critical pt.

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
x^2	+	+	+
$2x^3-2$	−	−	+
$f'(x)$	−	−	+
$f(x)$	\	\	/

f is increasing on $(1, \infty)$

f is decreasing on $(-\infty, 0) \cup (0, 1)$

local min at 1 : $f(1) = 1 + \frac{2}{1} = 3$

$f''(x) = 2 + 4x^{-3} = 2 + \frac{4}{x^3} = \frac{2x^3 + 4}{x^3} = 2\left(\frac{x^3 + 2}{x^3}\right)$

equal zero when $x = -\sqrt[3]{2}$

	$(-\infty, -\sqrt[3]{2})$	$(-\sqrt[3]{2}, 0)$	$(0, \infty)$
2	+	+	+
x^3	−	−	+
x^3+2	−	+	+
$f''(x)$	+	−	+

f is concave up on $(-\infty, -\sqrt[3]{2}) \cup (0, \infty)$

f is concave down on $(-\sqrt[3]{2}, 0)$

inflection pt when $x = -\sqrt[3]{2}$: $f(-\sqrt[3]{2}) = 0$

$f(x) = x^2 + \frac{2}{x} = \frac{x^3 + 2}{x}$

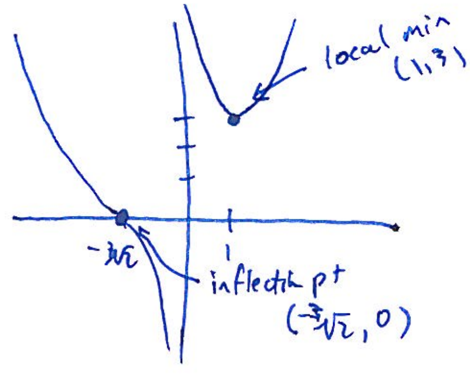
$0^3 + 2 \neq 0$

$0 = 0$

we have vertical asymptote $x = 0$.

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$



(52) $f(x) = y = x(\ln x)^2$ domain $(0, \infty)$

$f'(x) = (\ln x)^2 + 2\cancel{x} \ln x \cdot \left(\frac{1}{\cancel{x}}\right)$

$= (\ln x)^2 + 2 \ln x$

$= \ln x (\ln x + 2)$

never undefined on domain

equal to zero when $\ln x + 2 = 0$ or $\ln x = 0$

$\ln x = -2$ $\quad x = 1$

$x = e^{-2}$

	$(0, e^{-2})$	$(e^{-2}, 1)$	$(1, \infty)$
$\ln x$	$-$	$-$	$+$
$\ln x + 2$	$-$	$+$	$+$
$f'(x)$	$+$	$-$	$+$
$f(x)$	$\nearrow$	$\searrow$	$\nearrow$

f is increasing on $(0, e^{-2}) \cup (1, \infty)$

f is decreasing on $(e^{-2}, 1)$

f has local max at e^{-2}: $f(e^{-2}) = e^{-2}(\ln(e^{-2}))^2$

$= e^{-2}(-2)^2$

$= 4e^{-2}$

f has local min at $x = 1$: $f(1) = 1 \cdot (\ln(1))^2$

$= 0$

$f''(x) = 2 \ln x \cdot \frac{1}{x} + \frac{2}{x}$

$= \frac{2}{x}(\ln x + 1)$

is zero when $\ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1}$

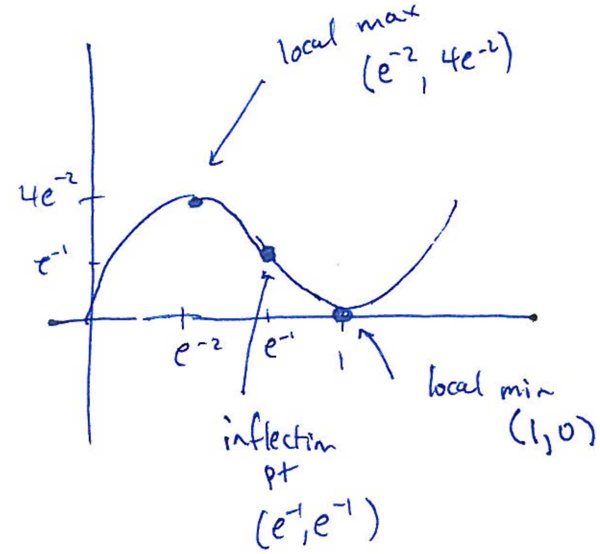
	$(0, e^{-1})$	(e^{-1}, ∞)
$\frac{2}{x}$	$+$	$+$
$\ln x + 1$	$-$	$+$
	$-$	$+$

f is concave down on $(0, e^{-1})$

f is concave up on (e^{-1}, ∞)

inflection pt when $x = e^{-1}$: $f(e^{-1}) = e^{-1}(\ln(-1))^2 = e^{-1}$

inflection pt (e^{-1}, e^{-1})



(58) $f(x) = \dfrac{e^x}{1+e^x}$ domain $\mathbb{R}$ (since $1+e^x$ is never equal to zero).

$$f'(x) = \frac{(1+e^x)e^x - e^x(e^x)}{(1+e^x)^2} = \frac{e^x(1+e^x-e^x)}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

e^x is never 0, , $(1+e^x)^2$ is never 0.

no critical pts.

$f'(x) > 0$ f is increasing on $\mathbb{R}$.

y-intercept: $\dfrac{e^0}{1+e^0} = \dfrac{1}{2}$

$$\cancel{f'(x) = \frac{e^x + 2e^{2x}}{(1+e^x)^2}}$$

$$\cancel{f''(x) = \frac{(1+e^x)^2(e^x+4e^{2x}) - (e^x+2e^{2x})(2(1+e^x)e^x)}{(1+e^x)^4}}$$

$$f'(x) = \frac{e^x}{(1+e^x)^2}$$

$$f''(x) = \frac{(1+e^x)^2 e^x - e^x(2(1+e^x))e^x}{(1+e^x)^4}$$

$$= \frac{(1+e^x)e^x(1+e^x - 2e^x)}{(1+e^x)^4}$$

$$= \frac{(1+e^x)(e^x)(1-e^x)}{(1+e^x)^4}$$

$1+e^x$ never 0, e^x never 0, $1-e^x$ is zero when $x=0$

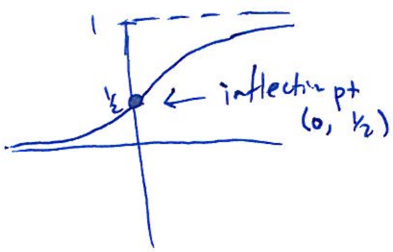
$f''(x)$ $\quad + \;|\; -$ (at 0)

f is concave up on $(-\infty, 0)$

f is concave down on $(0, \infty)$

inflection pt at $x=0$:

$$f(0) = \frac{1}{1+1} = \frac{1}{2}$$



$$\lim_{x\to\infty} \frac{e^x}{1+e^x} = \lim_{x\to\infty} 1 - \frac{1}{1+e^x} = 1.$$

$$\lim_{x\to-\infty} \frac{e^x}{1+e^x} = \lim_{x\to-\infty} 1 - \frac{1}{1+e^x} = 0.$$

(60) $y' = x^2 - x - 6 = (x-3)(x+2)$

$y'' = 2x - 1$

$y = f(x)$

critical pts $3, -2$

f is increasing on $(-\infty, -2) \cup (3, \infty)$

f is decreasing on $(-2, 3)$

	$(-\infty,-2)$	$(-2,3)$	$(3,\infty)$
$x-3$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
$f'(x)$	$+$	$-$	$+$
	/	\	/

$f''(x) = 2x - 1$

	$(-\infty, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
$2x-1$	$-$	$+$

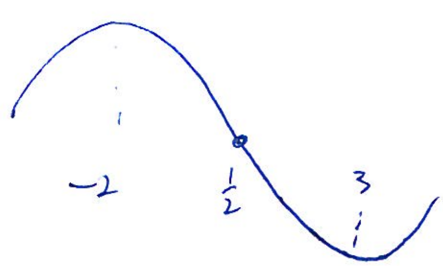
f is concave up on $(\frac{1}{2}, \infty)$

concave down on $(-\infty, \frac{1}{2})$

Critical pt at $x = \frac{1}{2}$

local max at $x = -2$:

local min at $x = 3$:



$y = f(x)$

62. $y' = x^2(2-x) = 2x^2 - x^3$

$y'' = 4x - 3x^2 = x(4-3x)$

critical pts $x = 0, 2$

		0		2	
$2-x$	+		+		−
x^2	+		+		+
	+		+		−

f is increasing on $(-\infty, 0) \cup (0, 2)$

f is decreasing on $(2, \infty)$

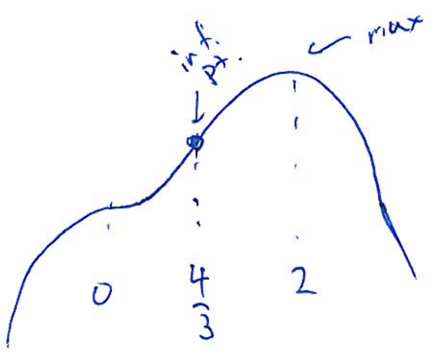
$y'' = 0$ when $x = 0$, $x = \frac{4}{3}$

		0		$4/3$	
x	−		+		+
$4-3x$	+		+		−
	−		+		−

f is concave down on $(-\infty, 0) \cup (4/3, \infty)$

f is concave up on $(0, 4/3)$

f has local max at 2:



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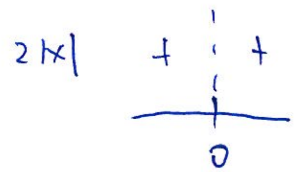
$$y' = 2|x| = \begin{cases} -2x & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases}$$

$$y'' = \begin{cases} -2 & \text{if } x < 0 \\ 2 & \text{if } x > 0 \end{cases}$$

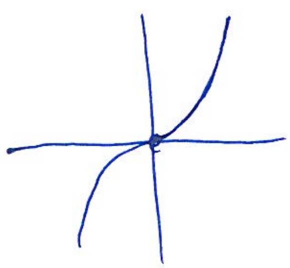
critical pt $x = 0$.



f is always increasing. no max, no min.

f is concave up on $(0, \infty)$, concave down on $(-\infty, 0)$

inflection pt when $x = 0$.



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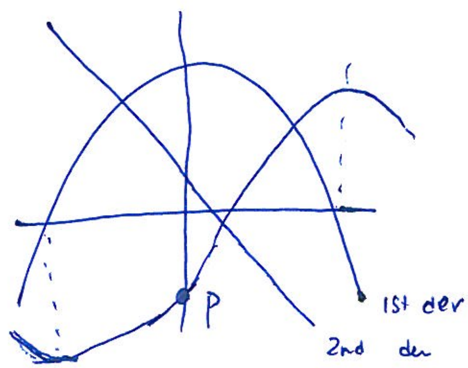


$y = f(x)$

(90) $y = \dfrac{x^2}{x^2-1} = \dfrac{x^2}{(x+1)(x-1)}$

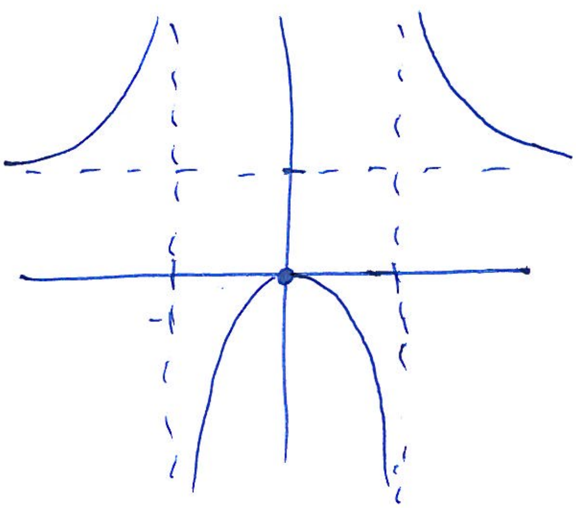
vertical asymptotes $x=1,\ x=-1$

$$\lim_{x \to \infty} \frac{x^2}{x^2-1} = 1$$

horizontal asymptote $y=1$

y - intercept 0

		-1		0		1	
x^2	+		+		+		+
$x+1$	−		+		+		+
$x-1$	−		−		−		+
	+		−		−		+



$$y' = \frac{(x^2-1)(2x) - x^2(2x)}{(x^2-1)^2}$$

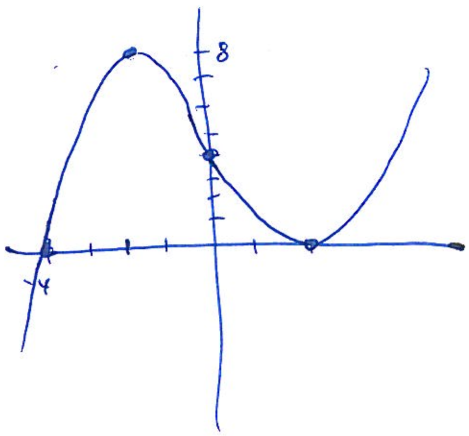
$$= \frac{-2x}{(x^2-1)^2}$$

y is increasing on $(-\infty,-1) \cup (-1,0)$

decreasing on $(0,1) \cup (1,\infty)$

		-1		0		1	
$-2x$	+		+		−		−
(x^2-1)	+		+		+		+
$f'(x)$	+		+		−		−

(104)



Curve contains pts

$(-2, 8)$, $(0, 4)$, $(2, 0)$

increasing on $(-\infty, -2) \cup (2, \infty)$

slope at 2 and -2 is 0.

decreasing on $(-2, 2)$

Concave down on $(-\infty, 0)$

Concave up on $(0, \infty)$

(106)

increasing on $(-\infty, 2) \cup (0, 2)$

decreasing on $(-2, 0) \cup (2, \infty)$

Concave down on $(-\infty, -1) \cup (1, \infty)$

Concave up on $(-1, 1)$

Contains pts

$(-2,2)$ $(-1,1)$ $(0,0)$

$(1,1)$ $(2,2)$.