4.7 SOLUTIONS/HINTS

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Recall that Newton's method produces a sequence $x_1, x_2, x_3, \dots$ of approximate solutions to $f(x) = 0$ given an initial guess x_0,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

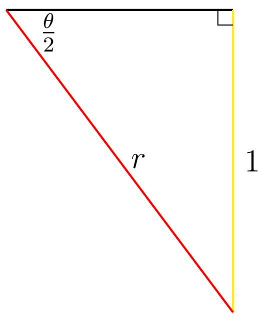
2. We compute $f'(x) = 3x^2 + 3$. Plugging $x_0 = 0$ into the formula for Newton's method, we get
$$x_1 = -\frac{1}{3}.$$
Plugging $x_1 = -1/3$ into the formula for Newton's method, we get
$$x_2 = -\frac{29}{90}.$$

4. We compute $f'(x) = 2 - 2x$ We plug x_0 into the formula to get x_1. We plug x_1 into the formula to get x_2. For $x_0 = 0$, we get
$$x_1 = -\frac{1}{2}, \quad \text{and} \quad x_2 = -\frac{5}{12}.$$
For $x_0 = 2$, we get
$$x_1 = \frac{5}{2}, \quad \text{and} \quad x_2 = \frac{29}{12}.$$

22. The graphs intersect when $\sqrt{x} = 3 - x^2$. Since Newton's method helps us approximate zeros, we need to rewrite this in the form $f(x) = 0$. Moving everything to one side, we see that we want the a solution to $f(x) = 0$, where $f(x) = 3 - x^2 - \sqrt{x}$. We compute that $f'(x) = -2x - \frac{1}{2}x^{-1/2}$. We then use Newton's method several times to get
$$\begin{aligned}
x_1 &= 1.44546136321895392547991604940\\
x_2 &= 1.35726980816019856304391663010\\
x_3 &= 1.35497934665600245042656257732\\
x_4 &= 1.35497780783011808206111183158\\
x_5 &= 1.35497780782942360206533157971\\
x_6 &= 1.35497780782942360206533143826\\
x_7 &= 1.35497780782942360206533143826\\
x_8 &= 1.35497780782942360206533143826\\
x_9 &= 1.35497780782942360206533143826\\
x_{10} &= 1.35497780782942360206533143826
\end{aligned}$$
This is pretty convincing evidence that the root is $r = 1.35498$, rounded to 5 decimal places.

30. For this exercise, use the picture from page 279. Note that from the definition of radians, we have that $\theta = \frac{s}{r} = \frac{3}{r}$. In particular, if we can estimate one, we have an estimate for the other.

The key point to notice is that if we bisect the angle θ, we get a line that is perpendicular to the yellow chord, which splits the yellow chord in half. Thus we get a triangle



It follows that $\sin(\frac{\theta}{2}) = \frac{1}{r}$. In particular, we have $\sin(\frac{\theta}{2}) = \frac{\theta}{3}$. To use Newton's method, we need to write this in the form $f(x) = 0$. Move everything to one side to see that we can take f to be

$$f(x) = \sin(\frac{x}{2}) - \frac{x}{3}.$$

Then $f'(x) = \cos(\frac{x}{2}) - \frac{1}{3}$. We compute several iterations of Newton's Method with initial guess $x_0 = 2$.

$$
\begin{aligned}
x_1 &= 4.7666711885629632923958688894 6\\
x_2 &= 3.4724486877147597617995150227 4\\
x_3 &= 3.0607406273086372034725360237 9\\
x_4 &= 2.9934708794411167088281880641 6\\
x_5 &= 2.9915646673971213756265792090 5\\
x_6 &= 2.9915631364451868864534954849 9\\
x_7 &= 2.9915631364441994204309881099 6\\
x_8 &= 2.9915631364441994204309876991 5\\
x_9 &= 2.9915631364441994204309876991 5\\
x_{10} &= 2.9915631364441994204309876991 5
\end{aligned}
$$

This is pretty convincing evidence that the solution, rounded to five decimal places is $\theta = 2.99156$. We have $r = \theta/3$, so we have $r = 0.99718$ rounded to five decimal places.

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