MATH 701: GRADUATE SEMINAR IN COMPUTATIONAL MATHEMATICS: REPRESENTATION THEORY (SPRING 2013)

Course number: MAT 701-01

Course title: Representation Theory

Credits: 3

Meetings: TBA

Prerequisites: Algebra, linear algebra, and basic topology

Instructor information:

Instructor: Dr. Dan Yasaki `d_yasaki@uncg.edu`
Homepage: `http://www.uncg.edu/mat/faculty/d_yasaki/teaching.html`
Office Hours (146 Petty): MWF 8:30–9:30 AM

For whom planned: Graduate students.

Catalog description: Readings from the literature of computational mathematics.

Student learning outcomes: Upon successful completion of this course students shall be able to

SLO 1: define basic terms associated with representation theory;
SLO 2: give examples of fundamental objects in representation theory;
SLO 3: explain definitions and give precise statements of important theorems of representation theory; and
SLO 4: construct and **defend** coherent mathematical proofs of statements in representation theory.

Teaching methods and assignments for achieving learning outcomes:

Reading: Weekly readings from the text and supplemental materials. (SLO 1)
Lectures: Weekly lectures will be presented by me and the student(s) on the material. (SLO 1–4)
Homework: Homework is the most important way to actually "learn" mathematics. This is the same sort of practice that is used to learn to play tennis or learn to play the piano. One cannot learn these things by watching them on television or reading about them in a book no more than one can learn mathematics by watching a lecture or reading a mathematics book. (SLO1–4)

Evaluation and grading: Semester averages are rounded to the nearest point, and letter grades are assigned on a 10 point scale.

A+ : 97–100	B+ : 87–89	C+ : 77–79	D+ : 67–69	
A : 93–96	B : 83–86	C : 73–76	D : 63–66	F : 0 – 59
A− : 90–92	B− : 80–82	C− : 70–72	D− : 60–62	

This document was last modified on January 10, 2014.

Homework: Homework will be collected weekly. All assignments are weighted equally. Homework counts for 20% of your final grade. (SLO 1–4)

Lectures: Weekly lectures presented by the student(s) on the material will be marked for accuracy and counts for 50% of your grade. (SLO 1–4)

Final paper: A final paper on some topic in representation theory will count for 30% of your grade. A list of possible topics will be provided. (SLO 1–4)

Required text:

William Fulton and Joe Harris, *Representation Theory: A First Course*, Springer-Verlag, 1991.

Academic Integrity Policy: Each student is required to sign the Academic Integrity Policy on all major work submitted for the course.

I have abided by the UNCG Academic Integrity Policy on this assignment.
Signature _____ Date _____

More information can be found at

`http://sa.uncg.edu/handbook/academic-integrity-policy/`.

Attendance Policy: Attendance is mandatory. Two consecutive absences or four total absences during the semester may result in a failing grade or removal from the course, regardless of semester average.

Additional information:

(1) Students with Disabilities: If you have a documented disability and wish to discuss academic accommodations, please contact me as soon as possible. You are responsible for contacting the OARS in 215 EUC (334-5440, `http://ods.dept.uncg.edu/`) and for arranging the necessary forms for me to fill out and sign. Without these forms the services provided by the OARS will not be available. OARS cannot schedule or reschedule tests without consent from the instructor.

(2) Assignments Policy:
   (a) Assignments are due at the beginning of class. Late assignments will be accepted as late as 5 PM on the due date for half credit and not accepted after that.
   (b) Written assignments must be
      (i) legible.
      (ii) stapled (if more than one page).
      (iii) not torn from a spiral bound notebook.

(3) Absence Policy: You are responsible for all missed material. Any missed assignment, test, or final exam will result in a score of 0. Make-up tests and final exam will be given only if you receive prior approval for a valid excuse by contacting me at least one week in advance.

(4) Copyright Policy: Selling or purchasing notes from classes for commercial gain is a violation of the UNCG Copyright Policy.

`http://policy.uncg.edu/copyright/`

Any student who sells notes taken in class for commercial gain, or who purchases notes taken by another student for commercial gain, is in violation of this policy and, by extension, is committing a violation of the Student Code of Conduct.

`http://sa.uncg.edu/handbook/student-code-of-conduct/`

(5) Email Policy: All email correspondence should be made using your UNCG email account. You must check your email regularly for updates and announcements.

TABLE 1. Topical outline

Sec.	Material covered
1.	**Representations of Finite Groups**
1.1	Definitions
1.2	Complete Reducibility; Schur's Lemma
1.3	Examples: Abelian Groups; $\mathcal{S}_3$
2.	**Characters**
2.1	Characters
2.2	The First Projection Formula and Its Consequences
2.3	Examples: $\mathcal{S}_4$ and $\mathcal{A}_4$
2.4	More Projection Formulas; More Consequences
3.	**Examples; Induced Representations; Group Algebras; Real Representations**
3.1	Examples: $\mathcal{S}_5$ and $\mathcal{A}_5$
3.2	Exterior Powers of the Standard Representation of $\mathcal{S}_d$
3.3	Induced Representations
3.4	The Group Algebra
3.5	Real Representations and Representations over Subfields of $\mathbb{C}$
7.	**Lie Groups**
7.1	Lie Groups: Definitions
7.2	Examples of Lie Groups
7.3	Two Constructions
8.	**Lie Algebras and Lie Groups**
8.1	Lie Algebras: Motivation and Definition
8.2	Examples of Lie Algebras
8.3	The Exponential Map
9.	**Initial Classification of Lie Algebras**
9.1	Rough Classification of Lie Algebras
9.2	Engel's Theorem and Lie's Theorem
9.3	Semisimple Lie Algebras
9.4	Simple Lie Algebras
10.	**Lie Algebras in Dimensions One, Two, and Three**
10.1	Dimensions One and Two
10.2	Dimension Three, Rank 1
10.3	Dimension Three, Rank 2
10.4	Dimension Three, Rank 3
11.	**Representations of $\mathfrak{sl}_2\mathbb{C}$**
11.1	The Irreducible Representations
11.2	A Little Plethysm
12.	**Representations of $\mathfrak{sl}_3\mathbb{C}$, Part I**
13.	**Representations of $\mathfrak{sl}_3\mathbb{C}$, Part II: Mainly Lots of Examples**
13.1	Examples
13.2	Description of the Irreducible Representations
13.3	A Little More Plethysm
14.	**The General Set-up: Analyzing the Structure and Representations of an Arbitrary Semisimple Lie Algebra**
14.1	Analyzing Simple Lie Algebras in General
14.2	About the Killing Form