# Calculus I to A Transition Test

## Review Problems

Try to solve the problems before looking at the solutions. Check your answers with the solutions. Learn from you mistakes.

Do not just study the answers.

Questions that are asked in a sentence should be answered with a sentence.

1. Define the *antiderivative* of a function $f$ on an interval $[a, b]$.
2. Find functions $f$ and $g$ with
$$\int f(x) \cdot g(x)\, dx \neq \int f(x)\, dx \cdot \int g(x)\, dx.$$
3. Solve the initial value problem $\frac{ds}{dt} = 12t(3t^2 - 1)$, $s(1) = 3$.
4. Solve the initial value problem $\frac{d^2y}{dx^2} = \frac{3x}{8}$, $y'(4) = 3$, $y(4) = 5$.
5. Find $\displaystyle\int 24x^3 - 20x^4 + 25x^5 + 9\, dx$.
6. Find $\displaystyle\int x^2 - 16x^3 + \frac{1}{3x^2}\, dx$.
7. Find $\displaystyle\int \frac{8x^8 + 6x^6 - 80}{x^6}\, dx$.
8. Find $\displaystyle\int 2e^x - 3e^{-2x}\, dx$.
9. Find $\displaystyle\int \frac{2}{\sqrt{x}}\, dx$.
10. Find $\displaystyle\int -\sin(x)\, dx$.

11. Using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base estimate the area under the graph of $f(x) = 8/x$ between $x = 1$ and $x = 33$ using

    (a) 2 rectangles of equal width

    (b) 4 rectangles of equal width

    Round to four decimal places.

12. Let $a$ be a positive real number. What is the area under the graph of $f(x) = 5$ between $x = 0$ and $x = a$ ?

13. Let $a$ be a positive real number. What is the area under the graph of $f(x) = 5x$ between $x = 0$ and $x = a$ ?

14. What is the area under the graph of $f(x) = 5x$ between $x = 1$ and $x = 7$ ?

15. Evaluate $\displaystyle\sum_{i=1}^{100} 3$.

16. Evaluate $\displaystyle\sum_{i=0}^{10} 2$.

17. Evaluate $\displaystyle\sum_{i=0}^{999} 2i$.

18. Evaluate $\displaystyle\sum_{k=0}^{3} \frac{k^2}{k+1}$.

19. Evaluate $\displaystyle\sum_{k=1}^{9} \left(k^3 + 2\right)$.

20. Let $f(x) = x^3$.

    (a) Give a formula for the approximation of the area under the curve $f$ between $x = 0$ and $x = 2$ in sigma notation. For your approximation use $n$ rectangles of equal width and use the right endpoint for the height.

    (b) Take the limit of your formula as $n \to \infty$.

    (c) What is the exact area under the curve ?

21. Give the definition of *Riemann sum* for a function $f$ on an interval $[a, b]$.

22. Define the definite integral of a function $f$ on the interval $[a, b]$.

23. Express the following limits as a definite integrals. The number $x_k^*$ is in the $k$-th subinterval of $P$ and $n$ is the number of intervals in $P$ and $\Delta x_k$ is the length of the $k$-th subinterval.

   (a) $\lim\limits_{||P||\to 0} \sum\limits_{k=1}^{n} \dfrac{1}{1+x_k^*} \Delta x_k$ where $P$ is a partition of $[2, 6]$.

   (b) $\lim\limits_{||P||\to 0} \sum\limits_{k=1}^{n} \dfrac{x_k^*}{4} \Delta x_k$ where $P$ is a partition of $[1, 3]$

   (c) $\lim\limits_{||P||\to 0} \sum\limits_{k=1}^{n} (x_k^*)^4 \Delta x_k$ where $P$ is a partition of $[-2, 4]$

   (d) $\lim\limits_{||P||\to 0} \sum\limits_{k=1}^{n} \sin^2(x_k^*) \Delta x_k$ where $P$ is a partition of $[0, \pi]$

24. Find a formula for the Riemann sum for $f$ obtained by dividing the given interval into $n$ subintervals of equal length and using the right endpoint for each $x_k^*$. Then take the limit of the sum as $n \to \infty$.

   (a) $f(x) = 4x$ on $[1, 3]$.
   (b) $f(x) = 6x^2$ on $[1, 3]$.

25. Find a formula for the Riemann sum for $f$ obtained by dividing the given interval into $n$ subintervals of equal length and using the right endpoint for each $x_k^*$. Then take the limit of the sum as $n \to \infty$.

   (a) $f(x) = x$ on $[a, b]$ where $a \in \mathbb{R}$ and $b \in \mathbb{R}$ with $a < b$.
   (b) $f(x) = x^2$ on $[a, b]$ where $a \in \mathbb{R}$ and $b \in \mathbb{R}$ with $a < b$.

   Why are these limits equal to $\displaystyle\int_a^b f(x)\,dx$ ?

26. Find a formula for the Riemann sum for $f(x) = x^2 - 1$ obtained by dividing $[0, 3]$ into $n$ subintervals of equal length and using right endpoints. Then take the limit of the sum as $n \to \infty$.

27. Interpret the following limit as the limit of a Riemann sum $\sum\limits_{k=1}^{n} f(x_k^*)\Delta x_k$ of some function $f$ with equally spaced sub-intervals of $[0, 1]$ using right endpoints:

$$L = \lim_{n\to\infty} \sum_{k=1}^{n} \left( \left(\frac{k}{n}\right)^4 + 3 \right) \left(\frac{1}{n}\right).$$

   Use the Fundamental Theorem of Calculus to find the exact value of the limit $L$.
   Fill in the blanks.

$$\begin{aligned}
\Delta x_k &= \underline{\hspace{4cm}} \\
x_k^* &= \underline{\hspace{4cm}} \\
f(x) &= \underline{\hspace{4cm}} \\
L &= \underline{\hspace{4cm}}
\end{aligned}$$

28. Suppose that $f$ and $h$ are integrable and that

$$\int_1^7 f(x)\,dx = -1, \quad \int_4^7 f(x)\,dx = 5, \quad \int_4^7 g(x)\,dx = 6.$$

Find

(a) $\displaystyle\int_1^7 -2f(x)\,dx$

(b) $\displaystyle\int_4^7 f(x) + 3g(x)\,dx$

(c) $\displaystyle\int_7^1 f(x)\,dx$

(d) $\displaystyle\int_1^4 f(x)\,dx$

29. Show that $\displaystyle\int_0^1 2\cos x\,dx$ cannot possibly be 3.

30. Show that $\displaystyle 0 \leq \int_0^5 (\sin x)^2\,dx \leq 5$.

31. State the mean value theorem for definite integrals.

32. State the fundamental theorem of calculus part I.

33. State the fundamental theorem of calculus part II.

34. Evaluate the integrals

(a) $\displaystyle\int_{-2}^2 x^3 - 2x + 3\,dx$

(b) $\displaystyle\int_0^\pi 1 + \cos x\,dx$

35. Find the average value of $f(x) = -5x^2 - 1$ on the interval $[0, 2]$.

36. Find the area between the curve $y = (x-2)(x-4)(x-6)$ on $[-1, 7]$ and the $x$-axis.

37. Find the area of the region between the curve $y = 3 - x^2$ and the line $y = -1$.

38. Let $f(x) = 1 - x^2$ and $g(x) = x - 1$. Find the area between the curves $y = f(x)$ and $y = g(x)$ between $x = -1$ and $x = 2$.